Rules for the Direction of the Mind

Translator's preface

Descartes' *Rules for the Direction of the Mind* (*Regulae ad Directionem Ingenii*) was written in Latin, probably in 1628 or a few years earlier, but was not published during the author's lifetime. A Dutch translation of the work appeared in Holland in 1684, and the first Latin edition was published in Amsterdam by P. and J. Blaeu in 1701.[1]

In the inventory of Descartes' papers made at Stockholm shortly after his death in 1650 the work is listed as 'Nine notebooks bound together, containing part of a Treatise on clear and useful Rules for the Direction of the Mind in the Search for Truth'. The original manuscript, which is lost, passed to Claude Clerselier, one of Descartes' staunchest supporters, who showed the work to several scholars, including Antoine Arnauld. The manuscript was seen also by Adrien Baillet, Descartes' biographer, who gave a summary of its contents in his *La Vie de Monsieur Des-Cartes* (1691). Leibniz bought a copy of the original manuscript in Amsterdam in 1670, and this copy has survived among the Leibniz papers in the Royal Public Library at Hanover.

The *Rules* was originally intended to contain three parts, each comprising twelve rules. The second set of twelve rules is incomplete, ending at Rule Twenty-one, and only the headings of Rules Nineteen to Twenty-one are given. The final set of twelve Rules is entirely missing; it appears that Descartes left this project unfinished. The first twelve Rules are concerned with simple propositions and the two cognitive operations by means of which they are known, intuition and deduction. The second set deal with what Descartes calls 'perfectly understood problems', i.e. problems in which the object of inquiry is a unique function of the data and which can be expressed in the forms of equations. Problems of this sort are confined largely to the sphere of mathematics. The projected third set of Rules would have dealt with 'imperfectly understood problems', i.e. problems which, owing to the multiplicity of the data involved, resist expression in the form of an equation; problems of this sort are prominent in the empirical sciences. Descartes had intended to

1 *R. Des-Cartes Opuscula posthuma, physica et mathematica.*

show how imperfectly understood problems can be reduced to perfectly understood ones.

The present translation is based primarily on the text in Volume x of Adam and Tannery.[1] There are differences of detail between the Amsterdam edition of 1701 and the Hanover manuscript; they were probably based on different copies of the original manuscript. Where the two texts differ, the 1701 edition in most cases provides the better reading, and Adam and Tannery generally follow this text. In several instances, however, readings other than those adopted by Adam and Tannery have been preferred in the present translation; these are described in footnotes when the variants are not given in Adam and Tannery, or when neither of the alternative variants yields an obviously preferable reading. The critical edition of Giovanni Crapulli[1] has been a useful supplement to Adam and Tannery, and several of Crapulli's readings have been adopted.

In the footnotes the Amsterdam edition of 1701 is referred to as A, and the Hanover manuscript as H.

D.M.

1 See General Introduction, p. x above.

2 *René Descartes: Regulae ad directionem ingenii: texte critique établi par Giovanni Crapulli avec la version hollandaise du XVIIème siècle* (The Hague: Martinus Nijhoff, 1966).

359

Rule One

The aim of our studies should be to direct the mind with a view to forming true and sound judgements about whatever comes before it.

Whenever people notice some similarity between two things, they are in the habit of ascribing to the one what they find true of the other, even when the two are not in that respect similar. Thus they wrongly compare the sciences, which consist wholly in knowledge acquired by the mind, with the arts, which require some bodily aptitude and practice. They recognize that one man cannot master all the arts at once and that it is easier to excel as a craftsman if one practises only one skill; for one man 360
cannot turn his hand to both farming and harp-playing, or to several different tasks of this kind, as easily as he can to just one of them. This has made people come to think that the same must be true of the sciences as well. Distinguishing the sciences by the differences in their objects, they think that each science should be studied separately, without regard to any of the others. But here they are surely mistaken. For the sciences as a whole are nothing other than human wisdom, which always remains one and the same, however different the subjects to which it is applied, it being no more altered by them than sunlight is by the variety of the things it shines on. Hence there is no need to impose any restrictions on our mental powers; for the knowledge of one truth does not, like skill in one art, hinder us from discovering another; on the contrary it helps us. Indeed, it seems strange to me that so many people should investigate with such diligence the virtues of plants,[1] the motions of the stars, the transmutations of metals, and the objects of similar disciplines, while hardly anyone gives a thought to good sense – to universal wisdom. For every other science is to be valued not so much for its own sake as for its contribution to universal wisdom. Hence, we have reason to propose this as our very first rule, since what makes us stray from the correct way of seeking the truth is chiefly our ignoring the general end of universal

1 The translation here follows the texts of A and H: AT, following an emendation by Leibniz, read 'the customs of men, the virtues of plants . . .'

wisdom and directing our studies towards some particular ends. I do not mean vile and despicable ends such as empty glory or base gain: specious arguments and tricks suited to vulgar minds clearly provide a much more
361 direct route to these ends than a sound knowledge of the truth could provide. I have in mind, rather, respectable and commendable ends, for these are often more subtly misleading – ends such as the pursuit of sciences conducive to the comforts of life or to the pleasure to be gained from contemplating the truth, which is practically the only happiness in this life that is complete and untroubled by any pain. We can indeed look forward to these legitimate fruits of the sciences; but if we think of them during our studies, they frequently cause us to overlook many items which are required for a knowledge of other things, because at first glance they seem of little use or of little interest. It must be acknowledged that all the sciences are so closely interconnected that it is much easier to learn them all together than to separate one from the other. If, therefore, someone seriously wishes to investigate the truth of things, he ought not to select one science in particular, for they are all interconnected and interdependent. He should, rather, consider simply how to increase the natural light of his reason, not with a view to solving this or that scholastic problem, but in order that his intellect should show his will what decision it ought to make in each of life's contingencies. He will soon be surprised to find that he has made far greater progress than those who devote themselves to particular studies, and that he has achieved not only everything that the specialists aim at but also goals far beyond any they can hope to reach.

362 *Rule Two*

We should attend only to those objects of which our minds seem capable of having certain and indubitable cognition.

All knowledge[1] is certain and evident cognition. Someone who has doubts about many things is no wiser than one who has never given them a thought; indeed, he appears less wise if he has formed a false opinion about any of them. Hence it is better never to study at all than to occupy ourselves with objects which are so difficult that we are unable to distinguish what is true from what is false, and are forced to take the doubtful as certain; for in such matters the risk of diminishing our knowledge is greater than our hope of increasing it. So, in accordance with this Rule, we reject all such merely probable cognition and resolve to believe only what is perfectly known and incapable of being doubted. Men of learning are perhaps convinced that there is very little indubitable

1 Lat. *scientia*, Descartes' term for systematic knowledge based on indubitable foundations.

knowledge, since, owing to a common human failing, they have disdained to reflect upon such indubitable truths, taking them to be too easy and obvious to everyone. But there are, I insist, a lot more of these truths than such people think – truths which suffice for the sure demonstration of countless propositions which so far they have managed to treat as no more than probable. Because they have thought it unbecoming for a man 363
of learning to admit to being ignorant on any matter, they have got so used to elaborating their contrived doctrines that they have gradually come to believe them and to pass them off as true.

Nevertheless, if we adhere strictly to this Rule, there will be very few things which we can get down to studying. For there is hardly any question in the sciences about which clever men have not frequently disagreed. But whenever two persons make opposite judgements about the same thing, it is certain that at least one of them is mistaken, and neither, it seems, has knowledge. For if the reasoning of one of them were certain and evident, he would be able to lay it before the other in such a way as eventually to convince his intellect as well. Therefore, concerning all such matters of probable opinion we can, I think, acquire no perfect knowledge, for it would be presumptuous to hope that we could gain more knowledge than others have managed to achieve. Accordingly, if my reckoning is correct, out of all the sciences so far devised, we are restricted to just arithmetic and geometry if we stick to this Rule.

Yet I do not wish on that account to condemn that method of philosophizing which others have hitherto devised, nor those weapons of the schoolmen, probable syllogisms,[1] which are just made for controversies. For these exercise the minds of the young, stimulating them with a certain rivalry; and it is much better that their minds should be informed with opinions of that sort – even though they are evidently 364
uncertain, being controversial among the learned – than that they should be left entirely to their own devices. Perhaps without guidance they might head towards a precipice, but so long as they follow in their masters' footsteps (though straying at times from the truth), they will surely hold to a course that is more secure, at least in the sense that it has already been tested by wiser heads. For our part, we are very glad that we had a scholastic education of this sort. But we are now freed from the oath which bound us to our master's words and are old enough to be no longer subject to the rod. So if we seriously wish to propose rules for ourselves which will help us scale the heights of human knowledge, we must include, as one of our primary rules, that we should take care not

1 I.e. syllogisms whose premisses are believed, but not known, to be true.

to waste our time by neglecting easy tasks and occupying ourselves only with difficult matters. That is just what many people do: they ingeniously construct the most subtle conjectures and plausible arguments on difficult questions, but after all their efforts they come to realize, too late, that rather than acquiring any knowledge, they have merely increased the number of their doubts.

Of all the sciences so far discovered, arithmetic and geometry alone are, as we said above, free from any taint of falsity or uncertainty. If we are to give a careful estimate of the reason why this should be so, we
365 should bear in mind that there are two ways of arriving at a knowledge of things – through experience and through deduction. Moreover, we must note that while our experiences of things are often deceptive, the deduction or pure inference of one thing from another can never be performed wrongly by an intellect which is in the least degree rational, though we may fail to make the inference if we do not see it. Furthermore, those chains with which dialecticians[1] suppose they regulate human reason seem to me to be of little use here, though I do not deny that they are very useful for other purposes. In fact none of the errors to which men – men, I say, not the brutes – are liable is ever due to faulty inference; they are due only to the fact that men take for granted certain poorly understood observations,[2] or lay down rash and groundless judgements.

These considerations make it obvious why arithmetic and geometry prove to be much more certain than other disciplines: they alone are concerned with an object so pure and simple that they make no assumptions that experience might render uncertain; they consist entirely in deducing conclusions by means of rational arguments. They are therefore the easiest and clearest of all the sciences and have just the sort of object we are looking for. Where these sciences are concerned it scarcely seems humanly possible to err, except through inadvertence. Yet we should not be surprised if many prefer of their own accord to apply their minds to other arts, or to philosophy. The reason for this is that everyone feels free to make more confident guesses about matters which
366 are obscure than about matters which are clear. It is much easier to hazard some conjecture on this or that question than to arrive at the exact truth about one particular question, however straightforward it may be.

Now the conclusion we should draw from these considerations is not that arithmetic and geometry are the only sciences worth studying, but rather that in seeking the right path of truth we ought to concern

1 Descartes' term for scholastic logic see below, *Principles*, p. 186.

2 Lat. *experimenta*; see footnote on the equivalent French term *expériences*, p. 143 below.

ourselves only with objects which admit of as much certainty as the demonstrations of arithmetic and geometry.

Rule Three

Concerning objects proposed for study, we ought to investigate what we can clearly and evidently intuit[1] *or deduce with certainty, and not what other people have thought or what we ourselves conjecture. For knowledge*[2] *can be attained in no other way.*

We ought to read the writings of the ancients, for it is of great advantage to be able to make use of the labours of so many men. We should do so both in order to learn what truths have already been discovered and also to be informed about the points which remain to be worked out in the various disciplines. But at the same time there is a considerable danger that if we study these works too closely traces of their errors will infect us and cling to us against our will and despite our precautions. For, once writers have credulously and heedlessly taken up a position on some controversial question, they are generally inclined to employ the most subtle arguments in an attempt to get us to adopt their point of view. On the other hand, whenever they have the luck to discover something
certain and evident, they always present it wrapped up in various 367
obscurities, either because they fear that the simplicity of their argument may depreciate the importance of their finding, or because they begrudge us the plain truth.

But even if all writers were sincere and open, and never tried to palm off doubtful matters as true, but instead put forward everything in good faith, we would always be uncertain which of them to believe, for hardly anything is said by one writer the contrary of which is not asserted by some other. It would be of no use to count heads, so as to follow the view which many authorities hold. For if the question at issue is a difficult one, it is more likely that few, rather than many, should have been able to discover the truth about it. But even if they all agreed among themselves, their teaching would still not be all we need. For example, even though we know other people's demonstrations by heart, we shall never become mathematicians if we lack the intellectual aptitude to solve any given problem. And even though we have read all the arguments of Plato and Aristotle, we shall never become philosophers if we are unable to make a sound judgement on matters which come up for discussion; in this case what we would seem to have learnt would not be science but history.

1 Lat. *intueri*, literally 'to look, gaze at'; used by Descartes as a technical term for immediate mental apprehension.

2 Lat. *scientia*; see footnote on p. 10 above.

Furthermore, we would be well-advised not to mix any conjectures into the judgements we make about the truth of things. It is most important to bear this point in mind. The main reason why we can find nothing in ordinary philosophy which is so evident and certain as to be
368 beyond dispute is that students of the subject first of all are not content to acknowledge what is clear and certain, but on the basis of merely probable conjectures venture also to make assertions on obscure matters about which nothing is known; they then gradually come to have complete faith in these assertions, indiscriminately mixing them up with others that are true and evident. The result is that the only conclusions they can draw are ones which apparently rest on some such obscure proposition, and which are accordingly uncertain.

But in case we in turn should slip into the same error, let us now review all the actions of the intellect by means of which we are able to arrive at a knowledge of things with no fear of being mistaken. We recognize only two: intuition and deduction.[1]

By 'intuition' I do not mean the fluctuating testimony of the senses or the deceptive judgement of the imagination as it botches things together, but the conception of a clear and attentive mind, which is so easy and distinct that there can be no room for doubt about what we are understanding. Alternatively, and this comes to the same thing, intuition is the indubitable conception of a clear and attentive mind which proceeds solely from the light of reason. Because it is simpler, it is more certain than deduction, though deduction, as we noted above, is not something a man can perform wrongly. Thus everyone can mentally intuit that he exists, that he is thinking, that a triangle is bounded by just three lines, and a sphere by a single surface, and the like. Perceptions such as these are more numerous than most people realize, disdaining as they do to turn their minds to such simple matters.

369 In case anyone should be troubled by my novel use of the term 'intuition' and of other terms to which I shall be forced to give a different meaning from their ordinary one, I wish to point out here that I am paying no attention to the way these terms have lately been used in the Schools. For it would be very difficult for me to employ the same terminology, when my own views are profoundly different. I shall take account only of the meanings in Latin of individual words and, when appropriate words are lacking, I shall use what seem the most suitable words, adapting them to my own meaning.

The self-evidence and certainty of intuition is required not only for apprehending single propositions, but also for any train of reasoning

1 *inductio* in A, almost certainly a misprint for *deductio*.

whatever. Take for example, the inference that 2 plus 2 equals 3 plus 1: not only must we intuitively perceive that 2 plus 2 make 4, and that 3 plus 1 make 4, but also that the original proposition follows necessarily from the other two.

There may be some doubt here about our reason for suggesting another mode of knowing in addition to intuition, *viz.* deduction, by which we mean the inference of something as following necessarily from some other propositions which are known with certainty. But this distinction had to be made, since very many facts which are not self-evident are known with certainty, provided they are inferred from true and known principles through a continuous and uninterrupted movement of thought in which each individual proposition is clearly intuited. This is similar to the way in which we know that the last link in
a long chain is connected to the first: even if we cannot take in at one 370
glance all the intermediate links on which the connection depends, we can have knowledge of the connection provided we survey the links one after the other, and keep in mind that each link from first to last is attached to its neighbour. Hence we are distinguishing mental intuition from certain deduction on the grounds that we are aware of a movement or a sort of sequence in the latter but not in the former, and also because immediate self-evidence is not required for deduction, as it is for intuition; deduction in a sense gets its certainty from memory. It follows that those propositions which are immediately inferred from first principles can be said to be known in one respect through intuition, and in another respect through deduction. But the first principles themselves are known only through intuition, and the remote conclusions only through deduction.

These two ways are the most certain routes to knowledge that we have. So far as our powers of understanding are concerned, we should admit no more than these and should reject all others as suspect and liable to error. This does not preclude our believing that what has been revealed by God is more certain than any knowledge, since faith in these matters, as in anything obscure, is an act of the will rather than an act of the understanding. And if our faith has a basis in our intellect, revealed truths above all can and should be discovered by one or other of the two ways we have just described, as we may show at greater length below.

Rule Four 371

We need a method if we are to investigate the truth of things.

So blind is the curiosity with which mortals are possessed that they often direct their minds down untrodden paths, in the groundless hope that they will chance upon what they are seeking, rather like someone who is

consumed with such a senseless desire to discover treasure that he continually roams the streets to see if he can find any that a passer-by might have dropped. This is how almost every chemist, most geometers, and many philosophers pursue their research. I am not denying that they sometimes are lucky enough in their wanderings to hit upon some truth, though on that account I rate them more fortunate than diligent. But it is far better never to contemplate investigating the truth about any matter than to do so without a method. For it is quite certain that such haphazard studies and obscure reflections blur the natural light and blind our intelligence. Those who are accustomed to walking in the dark weaken their eye-sight, the result being that they can no longer bear to be in broad daylight. Experience confirms this, for we very often find that people who have never devoted their time to learned studies make sounder and clearer judgements on matters which arise than those who have spent all their time in the Schools. By 'a method' I mean reliable
372 rules which are easy to apply, and such that if one follows them exactly, one will never take what is false to be true or fruitlessly expend one's mental efforts, but will gradually and constantly increase one's knowledge[1] till one arrives at a true understanding of everything within one's capacity.

There are two points here which we should keep in mind: we should never assume to be true anything which is false; and our goal should be to attain knowledge of all things. For, if we do not know something we are capable of knowing, this is simply because we have never discovered a way that might lead us to such knowledge, or because we have fallen into the opposite error.[2] But if our method properly explains how we should use our mental intuition to avoid falling into the opposite error and how we should go about finding the deductive inferences that will help us attain this all-embracing knowledge, then I do not see that anything more is needed to make it complete; for as I have already said, we can have no knowledge[1] without mental intuition or deduction. The method cannot go so far as to teach us how to perform the actual operations of intuition and deduction, since these are the simplest of all and quite basic. If our intellect were not already able to perform them, it would not comprehend any of the rules of the method, however easy they might be. As for other mental operations which dialectic[3] claims to direct with the help of those already mentioned, they are of no use here, or rather should be
373 reckoned a positive hindrance, for nothing can be added to the clear light of reason which does not in some way dim it.

1 Lat. *scientia*; see footnote on p. 10 above.
2 I.e. rejecting what is true through undue scepticism.
3 See footnote on p. 12 above.

So useful is this method that without it the pursuit of learning would, I think, be more harmful than profitable. Hence I can readily believe that the great minds of the past were to some extent aware of it, guided to it even by nature alone. For the human mind has within it a sort of spark of the divine, in which the first seeds of useful ways of thinking are sown, seeds which, however neglected and stifled by studies which impede them, often bear fruit of their own accord. This is our experience in the simplest of sciences, arithmetic and geometry: we are well aware that the geometers of antiquity employed a sort of analysis which they went on to apply to the solution of every problem, though they begrudged revealing it to posterity. At the present time a sort of arithmetic called 'algebra' is flourishing, and this is achieving for numbers what the ancients did for figures. These two disciplines are simply the spontaneous fruits which have sprung from the innate principles of this method. I am not surprised that, where the simplest objects of these disciplines are concerned, there has been a richer harvest of such fruits than in other disciplines in which greater obstacles tend to stifle progress. But no doubt these too could achieve a perfect maturity if only they were cultivated with extreme care.

That is in fact what I have principally aimed at achieving in this treatise. I would not value these Rules so highly if they were good only for solving those pointless problems with which arithmeticians and geometers are inclined to while away their time, for in that case all I could credit myself with achieving would be to dabble in trifles with
greater subtlety than they. I shall have much to say below about figures 374
and numbers, for no other disciplines can yield illustrations as evident and certain as these. But if one attends closely to my meaning, one will readily see that ordinary mathematics is far from my mind here, that it is quite another discipline I am expounding, and that these illustrations are more its outer garments than its inner parts. This discipline should contain the primary rudiments of human reason and extend to the discovery of truths in any field whatever. Frankly speaking, I am convinced that it is a more powerful instrument of knowledge than any other with which human beings are endowed, as it is the source of all the rest. I have spoken of its 'outer garment', not because I wish to conceal this science and shroud it from the gaze of the public; I wish rather to clothe and adorn it so as to make it easier to present to the human mind.

When I first applied my mind to the mathematical disciplines, I at once read most of the customary lore which mathematical writers pass on to us. I paid special attention to arithmetic and geometry, for these were
said to be the simplest and, as it were, to lead into the rest. But in neither 375
subject did I come across writers who fully satisfied me. I read much about numbers which I found to be true once I had gone over the

calculations for myself. The writers displayed many geometrical truths before my very eyes, as it were, and derived them by means of logical arguments. But they did not seem to make it sufficiently clear to my mind why these things should be so and how they were discovered. So I was not surprised to find that even many clever and learned men, after dipping into these arts, either quickly lay them aside as childish and pointless or else take them to be so very difficult and complicated that they are put off at the outset from learning them. For there is really nothing more futile than so busying ourselves with bare numbers and imaginary figures that we seem to rest content in the knowledge of such trifles. And there is nothing more futile than devoting our energies to those superficial proofs which are discovered more through chance than method and which have more to do with our eyes and imagination than our intellect; for the outcome of this is that, in a way, we get out of the habit of using our reason. At the same time there is nothing more complicated than using such a method of proof to resolve new problems which are beset with numerical disorder. Later on I wondered why the founders of philosophy would admit no one to the pursuit of wisdom who was unversed in mathematics[1] – as if they thought that this
376 discipline was the easiest and most indispensable of all for cultivating and preparing the mind to grasp other more important sciences. I came to suspect that they were familiar with a kind of mathematics quite different from the one which prevails today; not that I thought they had a perfect knowledge of it, for their wild exultations and thanksgivings for trivial discoveries clearly show how rudimentary their knowledge must have been. I am not shaken in this opinion by those machines[2] of theirs which are so much praised by historians. These mechanical devices may well have been quite simple, even though the ignorant and wonder-loving masses may have raised them to the level of marvels. But I am convinced that certain primary seeds of truth naturally implanted in human minds thrived vigorously in that unsophisticated and innocent age – seeds which have been stifled in us through our constantly reading and hearing all sorts of errors. So the same light of the mind which enabled them to see (albeit without knowing why) that virtue is preferable to pleasure, the good preferable to the useful, also enabled them to grasp true ideas in philosophy and mathematics, although they were not yet able fully to master such sciences. Indeed, one can even see some traces of this true mathematics, I think, in Pappus and Diophantus[3] who, though not of

1 A reference to Plato's Academy, over the entrance to which was inscribed the motto, 'No one ignorant of geometry may enter.'
2 Perhaps an allusion to mechanical devices such as the wooden dove (which could fly) constructed by Archytas of Tarentum, a friend of Plato.
3 Greek mathematicians working in Alexandria in the third century A.D.

that earliest antiquity, lived many centuries before our time. But I have come to think that these writers themselves, with a kind of pernicious cunning, later suppressed this mathematics as, notoriously, many inventors are known to have done where their own discoveries were concerned. They may have feared that their method, just because it was
so easy and simple, would be depreciated if it were divulged; so to gain our 377
admiration, they may have shown us, as the fruits of their method, some barren truths proved by clever arguments, instead of teaching us the method itself, which might have dispelled our admiration. In the present age some very gifted men have tried to revive this method, for the method seems to me to be none other than the art which goes by the outlandish name of 'algebra' – or at least it would be if algebra were divested of the multiplicity of numbers and incomprehensible figures which overwhelm it and instead possessed that abundance of clarity and simplicity which I believe the true mathematics ought to have. It was these thoughts which made me turn from the particular studies of arithmetic and geometry to a general investigation of mathematics. I began my investigation by inquiring what exactly is generally meant by the term 'mathematics'[1] and why it is that, in addition to arithmetic and geometry, sciences such as astronomy, music, optics, mechanics, among others, are called branches of mathematics. To answer this it is not enough just to look at the etymology of the word, for, since the word 'mathematics' has the same meaning as 'discipline',[2] these subjects have as much right to be called 'mathematics' as geometry has. Yet it is evident that almost anyone with the slightest education can easily tell the difference in any context between what relates to mathematics and what to the other disciplines. When I considered the matter more closely, I came to see that the
exclusive concern of mathematics is with questions of order or measure 378
and that it is irrelevant whether the measure in question involves numbers, shapes, stars, sounds, or any other object whatever. This made me realize that there must be a general science which explains all the points that can be raised concerning order and measure irrespective of the subject-matter, and that this science should be termed *mathesis universalis*[3] – a venerable term with a well-established meaning – for it covers everything that entitles these other sciences to be called branches of mathematics. How superior it is to these subordinate sciences both in utility and simplicity is clear from the fact that it covers all they deal with, and more besides; and any difficulties it involves apply to these as well, whereas their particular subject-matter involves difficulties which it lacks. Now everyone knows the name of this subject and without even

1 Descartes uses the term *mathesis*, from the Greek, *μάθησις*, literally 'learning'.
2 Lat. *disciplina*, from *discere*, 'to learn'.
3 I.e. 'universal mathematics'.

studying it understands what its subject-matter is. So why is it that most people painstakingly pursue the other disciplines which depend on it, and no one bothers to learn this one? No doubt I would find that very surprising if I did not know that everyone thinks the subject too easy, and if I had not long since observed that the human intellect always bypasses subjects which it thinks it can easily master and directly hurries on to new and grander things.

Aware how slender my powers are, I have resolved in my search for
379 knowledge of things to adhere unswervingly to a definite order, always starting with the simplest and easiest things and never going beyond them till there seems to be nothing further which is worth achieving where they are concerned. Up to now, therefore, I have devoted all my energies to this universal mathematics, so that I think I shall be able in due course to tackle the somewhat more advanced sciences, without my efforts being premature. But before I embark on this task I shall try to bring together and arrange in an orderly manner whatever I thought noteworthy in my previous studies, so that when old age dims my memory I can readily recall it hereafter, if I need to, by consulting this book, and so that, having disburdened my memory, I can henceforth devote my mind more freely to what remains.

Rule Five

The whole method consists entirely in the ordering and arranging of the objects on which we must concentrate our mind's eye if we are to discover some truth. We shall be following this method exactly if we first reduce complicated and obscure propositions step by step to simpler ones, and then, starting with the intuition of the simplest ones of all, try to ascend through the same steps to a knowledge of all the rest.

This one Rule covers the most essential points in the whole of human endeavour. Anyone who sets out in quest of knowledge of things must
380 follow this Rule as closely as he would the thread of Theseus if he were to enter the Labyrinth. But many people either do not reflect upon what the Rule prescribes, or ignore it altogether, or presume that they have no need of it. They frequently examine difficult problems in a very disorderly manner, behaving in my view as if they were trying to get from the bottom to the top of a building at one bound, spurning or failing to notice the stairs designed for that purpose. Astrologers all do likewise: they do not know the nature of the heavens and do not even make any accurate observations of celestial motions, yet they expect to be able to delineate the effects of these motions. So too do most of those who study mechanics apart from physics and, without any proper plan, construct

new instruments for producing motion. This applies also to those philosophers who take no account of experience and think that truth will spring from their brains like Minerva from the head of Jupiter.

All those just mentioned are plainly violating this Rule. But the order that is required here is often so obscure and complicated that not everyone can make out what it is; hence it is virtually impossible to guard against going astray unless one carefully observes the message of the following Rule.

Rule Six 381

In order to distinguish the simplest things from those that are complicated and to set them out in an orderly manner, we should attend to what is most simple in each series of things in which we have directly deduced some truths from others, and should observe how all the rest are more, or less, or equally removed from the simplest.

Although the message of this Rule may not seem very novel, it contains nevertheless the main secret of my method; and there is no more useful Rule in this whole treatise. For it instructs us that all things can be arranged serially in various groups, not in so far as they can be referred to some ontological genus (such as the categories into which philosophers divide things[1]), but in so far as some things can be known on the basis of others. Thus when a difficulty arises, we can see at once whether it will be worth looking at any others first, and if so which ones and in what order.

In order to be able to do this correctly, we should note first that everything, with regard to its possible usefulness to our project, may be termed either 'absolute' or 'relative' – our project being, not to inspect the isolated natures of things, but to compare them with each other so that some may be known on the basis of others.

I call 'absolute' whatever has within it the pure and simple nature in question; that is, whatever is viewed as being independent, a cause, simple, universal, single, equal, similar, straight, and other qualities of that sort. I call this the simplest and the easiest thing when we can make 382
use of it in solving problems.

The 'relative', on the other hand, is what shares the same nature, or at least something of the same nature, in virtue of which we can relate it to the absolute and deduce it from the absolute in a definite series of steps. The concept of the 'relative' involves other terms besides, which I call 'relations': these include whatever is said to be dependent, an effect, composite, particular, many, unequal, dissimilar, oblique, etc. The further

1 For example, the Aristotelian categories of substance, quality, quantity, relation, etc.

removed from the absolute such relative attributes are, the more mutually dependent relations of this sort they contain. This Rule points out that all these relations should be distinguished, and the interconnections between them, and their natural order, should be noted, so that given the last term we should be able to reach the one that is absolute in the highest degree, by passing through all the intermediate ones.

The secret of this technique consists entirely in our attentively noting in all things that which is absolute in the highest degree. For some things are more absolute than others from one point of view, yet more relative from a different point of view. For example, the universal is more absolute than the particular, in virtue of its having a simpler nature, but it can also be said to be more relative than the particular in that it depends upon particulars for its existence, etc. Again, certain things sometimes are really more absolute than others, yet not the most absolute of all. Thus a species is something absolute with respect to particulars, but with respect to the genus it is relative; and where measurable items are concerned, extension is something absolute, but among the varieties of extension
383 length is something absolute, etc. Furthermore, in order to make it clear that what we are contemplating here is the series of things to be discovered, and not the nature of each of them, we have deliberately listed 'cause' and 'equal' among the absolutes, although their nature really is relative. Philosophers, of course, recognize that cause and effect are correlatives; but in the present case, if we want to know what the effect is, we must know the cause first, and not *vice versa*. Again, equals are correlative with one another, but we can know what things are unequal only by comparison with equals, and not *vice versa*, etc.

Secondly, we should note that there are very few pure and simple natures which we can intuit straight off and *per se* (independently of any others) either in our sensory experience or by means of a light innate within us. We should, as I said, attend carefully to the simple natures which can be intuited in this way, for these are the ones which in each series we term simple in the highest degree. As for all the other natures, we can apprehend them only by deducing them from those which are simple in the highest degree, either immediately and directly, or by means of two or three or more separate inferences. In the latter case we should also note the number of these inferences so that we may know whether the separation between the conclusion and the primary and supremely simple proposition is by way of a greater or fewer number of steps. And the chain of inferences – which gives rise to those series of objects of investigation to which every problem must be reduced – is such throughout that the problem can be investigated by a reliable method.
384 But since it is not easy to review all the connections together, and

moreover, since our task is not so much to retain them in our memory as to distinguish them with, as it were, the sharp edge of our mind, we must seek a means of developing our intelligence in such a way that we can discern these connections immediately whenever the need arises. In my experience there is no better way of doing this than by accustoming ourselves to reflecting with some discernment on the minute details of the things we have already perceived.

The third and last point is that we should not begin our studies by investigating difficult matters. Before tackling any specific problems we ought first to make a random selection of truths which happen to be at hand, and ought then to see whether we can deduce some other truths from them step by step, and from these still others, and so on in logical sequence. This done, we should reflect attentively on the truths we have discovered and carefully consider why it was we were able to discover some of these truths sooner and more easily than others, and what these truths are. This will enable us to judge, when tackling a specific problem, what points we may usefully concentrate on discovering first. For example, say the thought occurs to me that the number 6 is twice 3: I may then ask what twice 6 is, *viz.* 12; I may, if I like, go on to ask what twice 12 is, *viz.* 24, and what twice 24 is, *viz.* 48, etc. It would then be easy for me to deduce that there is the same ratio between 3 and 6 as between 6 and 12, and again the same ratio between 12 and 24, etc., and hence that the numbers 3, 6, 12, 24, 48, etc. are continued proportionals. All of this is so clear as to seem almost childish; nevertheless when I think carefully
about it, I can see what sort of complications are involved in all the 385
questions one can ask about the proportions or relations between things, and in what order the questions should be investigated. This one point encompasses the essential core of the entire science of pure mathematics.

For I notice first that it was no more difficult to discover what twice 6 is than twice 3, and that whenever we find a ratio between any two magnitudes we can always find, just as easily, innumerable others which have the same ratio between them. The nature of the problem is no different when we are trying to find three, four, or more magnitudes of this sort, since each one has to be found separately and without regard to the others. I next observe that given the magnitudes 3 and 6, I easily found[1] a third magnitude which is in continued proportion, *viz.* 12, yet, when the extreme terms 3 and 12 were given, I could not find just as easily the mean proportional, 6. If we look into the reason for this, it is obvious that we have here a quite different type of problem from the preceding one. For, if we are to find the mean proportional, we must attend at the

1 Reading *invenerim*, A (following Crapulli) rather than *inveneris* ('you found'), H and AT.

same time to the two extreme terms and the ratio between them, in order to obtain a new ratio by dividing this one.[1] This is a very different task from that of finding a third magnitude, given two magnitudes in continued proportion.[2] I can go even further and ask whether, given the numbers 3 and 24, it would be just as easy to find one of the two mean
386 proportionals, *viz.* 6 and 12. Here we have another sort of problem again, an even more complicated one than either of the preceding ones. We have to attend not just to one thing or to two but to three different things at the same time, if we are to find a fourth.[3] We can go even further and see whether, given just 3 and 48, it would be still more difficult to find one of the three mean proportionals, *viz.* 6, 12 and 24. At first sight it does indeed seem to be more difficult. But then the thought immediately strikes us that this problem can be split up and made easier: first we look for the single mean proportional between 3 and 48, *viz.* 12; then we look for a further mean proportional between 3 and 12, *viz.* 6; then another between 12 and 48, *viz.* 24. In that way we reduce the problem to one of the second kind described above.

Moreover, from these examples I realize how in our pursuit of knowledge of a given thing we can follow different paths, one of which is much more difficult and obscure than the other. If, for example, we are asked to find the four proportionals, 3, 6, 12, 24, given any two consecutive members of the series, such as 3 and 6, or 6 and 12, or 12 and 24, it will be a very easy task to find the others. In this case we shall say that the proposition we are seeking is investigated in a direct way. But if two alternate numbers are given, such as 3 and 12, or 6 and 24, and we are to work out the others from these, in that case we shall say that the problem is investigated indirectly by the first method. Likewise, if we are to find the intermediate numbers, 6 and 12, given the two extremes, 3
387 and 24, then the problem will be investigated indirectly by the second method. I could thus go on even further and draw many other conclusions from this one example. But these points will suffice to enable the reader to see what I mean when I say that some proposition is deduced 'directly' or 'indirectly', and will suffice to make him bear in mind that on the basis of our knowledge of the most simple and primary things we can make many discoveries, even in other disciplines, through careful reflection and discriminating inquiry.

1 The problem: to find an x such that $3/x = x/12$.
2 The problem: to find an x such that $3/6 = 6/x$.
3 The problem: to find an x and y such that $3/x = x/y = y/24$.

Rule Seven

In order to make our knowledge[1] complete, every single thing relating to our undertaking must be surveyed in a continuous and wholly uninterrupted sweep of thought, and be included in a sufficient and well-ordered enumeration.

It is necessary to observe the points proposed in this Rule if we are to admit as certain those truths which, we said above, are not deduced immediately from first and self-evident principles. For this deduction sometimes requires such a long chain of inferences that when we arrive at such a truth it is not easy to recall the entire route which led us to it. That is why we say that a continuous movement of thought is needed to make good any weakness of memory. If, for example, by way of separate operations, I have come to know first what the relation between the magnitudes A and B is, and then between B and C, and between C and D, and finally between D and E, that does not entail my seeing what the relation is between A and E; and I cannot grasp what the relation is just 388
from those I already know, unless I recall all of them. So I shall run through them several times in a continuous movement of the imagination, simultaneously intuiting one relation and passing on to the next, until I have learnt to pass from the first to the last so swiftly that memory is left with practically no role to play, and I seem to intuit the whole thing at once. In this way our memory is relieved, the sluggishness of our intelligence redressed, and its capacity in some way enlarged.

In addition, this movement must nowhere be interrupted. Frequently those who attempt to deduce something too swiftly and from remote initial premisses do not go over the entire chain of intermediate conclusions very carefully, but pass over many of the steps without due consideration. But, whenever even the smallest link is overlooked the chain is immediately broken, and the certainty of the conclusion entirely collapses.

We maintain furthermore that enumeration is required for the completion of our knowledge.[1] The other Rules do indeed help us resolve most questions, but it is only with the aid of enumeration that we are able to make a true and certain judgement about whatever we apply our minds to. By means of enumeration nothing will wholly escape us and we shall be seen to have some knowledge on every question.

In this context enumeration,[2] or induction, consists in a thorough investigation of all the points relating to the problem at hand, an investigation which is so careful and accurate that we may conclude with

1 Lat. *scientia*; see footnote on p. 10 above.
2 Reading *hic*, A, H (following Crapulli), rather than *haec* ('This enumeration'), AT.

manifest certainty that we have not inadvertently overlooked anything.
389 So even though the object of our inquiry eludes us, provided we have made an enumeration we shall be wiser at least to the extent that we shall perceive with certainty that it could not possibly be discovered by any method known to us. If we have managed to examine all the humanly accessible paths towards the object of our inquiry (which we often do), we shall be entitled confidently to assert that knowledge of it lies wholly beyond the reach of the human mind.

We should note, moreover, that by 'sufficient enumeration' or 'induction' we just mean the kind of enumeration which renders the truth of our conclusions more certain than any other kind of proof (simple intuition excepted) allows. But when our knowledge of something is not reducible to simple intuition and we have cast off our syllogistic fetters, we are left with this one path, which we should stick to with complete confidence. For if we have deduced one fact from another immediately, then provided the inference is evident, it already comes under the heading of true intuition. If on the other hand we infer a proposition from many disconnected propositions, our intellectual capacity is often insufficient to enable us to encompass all of them in a single intuition; in which case we must be content with the level of certainty which the above operation allows. In the same way, our eyes cannot distinguish at one glance all the links in a very long chain; but, if we have seen the connections between each link and its neighbour, this enables us to say that we have seen how the last link is connected with the first.

I said that this operation should be 'sufficient', because it can often be deficient and hence liable to error. For sometimes, even though we survey
390 many points in our enumeration which are quite evident, yet if we make even the slightest omission, the chain is broken and the certainty of the conclusion is entirely lost. Again, sometimes we do cover everything in our enumeration, yet fail to distinguish one thing from another, so that our knowledge of them all is simply confused.

The enumeration should sometimes be complete, and sometimes distinct, though there are times when it need be neither. That is why I said only that the enumeration must be sufficient. For if I wish to determine by enumeration how many kinds of corporeal entity there are or how many are in some way perceivable by the senses, I shall not assert that there are just so many and no more, unless I have previously made sure I have included them all in my enumeration and have distinguished one from another. But if I wish to show in the same way that the rational soul is not corporeal, there is no need for the enumeration to be complete; it will be sufficient if I group all bodies together into several classes so as to demonstrate that the rational soul

cannot be assigned to any of these. To give one last example, say I wish to show by enumeration that the area of a circle is greater than the area of any other geometrical figure whose perimeter is the same length as the circle's. I need not review every geometrical figure. If I can demonstrate that this fact holds for some particular figures, I shall be entitled to conclude by induction[1] that the same holds true in all the other cases as well.

I said also that the enumeration must be well-ordered, partly because there is no more effective remedy for the defects I have just listed than a well-ordered scrutiny of all the relevant items, and partly because, if every single thing relevant to the question in hand were to be separately 391
scrutinized, one lifetime would generally be insufficient for the task, for either there would be too many such things or the same things would keep cropping up. But if we arrange all of the relevant items in the best order, so that for the most part they fall under definite classes, it will be sufficient if we look closely at one class, or at a member of each particular class, or at some classes rather than others. If we do that, we shall at any rate never pointlessly go over the same ground twice, and thanks to our well-devised order, we shall often manage to review quickly and effortlessly a large number of items which at first sight seemed formidably large.

In such cases the order in which things are enumerated can usually be varied; it is a matter for individual choice. For that reason, if our choice is to be intelligently thought out we should bear in mind what was said in Rule Five. In the more frivolous of man's skills there are many things whose method of invention consists entirely in arranging things in this orderly way. Thus if you want to construct a perfect anagram by transposing the letters of a name, there is no need to pass from the very easy to the more difficult, nor to distinguish what is absolute from what is relative, for these operations have no place here. All you need do is to decide on an order for examining permutations of letters so that you never go over the same permutations twice. The number of these permutations should, for example, be arranged into definite classes, so that it becomes immediately obvious which ones present the greater prospect of finding what you are looking for. If this is done, the task will seldom be tedious; it will be mere child's play.

Now, these last three Rules should not be separated. We should 392
generally think of them together, since they all contribute equally to the perfection of the method. It was immaterial which of them we expounded first. We are giving only a brief account of them here, for our task in

1 'Induction' here seems to have its standard sense of 'inference from particular instances of something to all instances'.

the remainder of the treatise will be confined almost entirely to explicating in detail what we have so far covered in general terms.

Rule Eight

If in the series of things to be examined we come across something which our intellect is unable to intuit sufficiently well, we must stop at that point, and refrain from the superfluous task of examining the remaining items.

The three preceding Rules prescribe and explain the order to be followed; the present Rule shows when order is absolutely necessary, and when it is merely useful. It is necessary that we examine whatever constitutes an integral step in the series through which we must pass when we proceed from relative terms to something absolute or *vice versa*, before considering all that follows in the series. Of course if many things belong to a given step, as is often the case, it is always useful to survey all of them in due order. But we are not forced to follow the order strictly and rigidly; generally we may proceed further, even although we do not have clear
393 knowledge of all the terms of the series, but only of a few or just one of them.

This Rule is a necessary consequence of the reasons I gave in support of Rule Two. But it should not be thought that this Rule contributes nothing new to the advancement of learning, even though it seems merely to deter us from discussing certain things and to bring no truth to light. Indeed, all it teaches beginners is that they should not waste their efforts, and it does so in practically the same manner as Rule Two. But it shows those who have perfectly mastered the preceding seven Rules how they can achieve for themselves, in any science whatever, results so satisfactory that there is nothing further they will desire to achieve. If anyone observes the above Rules exactly when trying to solve some problem or other, but is instructed by the present Rule to stop at a certain point, he will know for sure that no amount of application will enable him to find the knowledge[1] he is seeking; and that not because of any defect of his intelligence, but because of the obstacle which the nature of the problem itself or the human condition presents. His recognition of this point is just as much knowledge[1] as that which reveals the nature of the thing itself; and it would, I think, be quite irrational if he were to stretch his curiosity any further.

Let us illustrate these points with one or two examples. If, say, someone whose studies are confined to mathematics tries to find the line
394 called the 'anaclastic' in optics[2] – the line from which parallel rays are so

1 Lat. *scientia*; see footnote on p. 10 above.
2 Descartes solved this problem in Discourse 8 of his *Optics*.

refracted that they intersect at a single point – he will easily see, by following Rules Five and Six, that the determination of this line depends on the ratio of the angles of refraction to the angles of incidence. But he will not be able to find out what this ratio is, since it has to do with physics rather than with mathematics. So he will be compelled to stop short right at the outset. If he proposes to learn it from the philosophers or derive it from experience, he will achieve nothing, for that would be to violate Rule Three. Besides, the problem before him is composite and relative; and it is possible to have experiential knowledge which is certain only of things which are entirely simple and absolute, as I shall show in the appropriate place. Again, it is no use his assuming some particular ratio between the angles in question, one he conjectures to be most likely the real one; for in that case what he was seeking to determine would no longer be the anaclastic – it would merely be the line which was the logical consequence of his supposition.

Now take someone whose studies are not confined to mathematics and who, following Rule One, eagerly seeks the truth on any question that arises: if he is faced with the same problem, he will discover when he goes into it that the ratio between the angles of incidence and the angles of refraction depends upon the changes in these angles brought about by differences in the media. He will see that these changes depend on the manner in which a ray passes through the entire transparent body,[1] and that knowledge of this process presupposes also a knowledge of the nature of the action of light. Lastly, he will see that to understand the 395
latter process he must know what a natural power in general is – this last being the most absolute term in this whole series. Once he has clearly ascertained this through mental intuition, he will, in accordance with Rule Five, retrace his course through the same steps. If, at the second step, he is unable to discern at once what the nature of light's action is, in accordance with Rule Seven he will make an enumeration of all the other natural powers, in the hope that a knowledge of some other natural power will help him understand this one, if only by way of analogy – but more of this later.[2] Having done that, he will investigate the way in which the ray passes through the whole transparent body. Thus he will follow up the remaining points in due order, until he arrives at the anaclastic itself. Even though the anaclastic has been the object of much fruitless research in the past, I can see nothing to prevent anyone who uses our method exactly from gaining a clear knowledge of it.

But let us take the finest example of all. If someone sets himself the

1 Lat. *totum diaphanum*, the very fluid 'subtle matter' which Descartes took to be the medium of the transmission of light. Cf. *Optics*, p. 154 below.

2 This topic is not discussed in the extant portions of the *Rules*. See however *Optics*, Discourses 1 and 2 (pp. 152–64 below).

problem of investigating every truth for the knowledge of which human reason is adequate – and this, I think, is something everyone who earnestly strives after good sense should do once in his life – he will indeed discover by means of the Rules we have proposed that nothing can be known prior to the intellect, since knowledge of everything else depends on the intellect, and not *vice versa*. Once he has surveyed everything that follows immediately upon knowledge of the pure intellect, among what remains he will enumerate whatever instruments of knowledge we possess in addition to the intellect; and there are only two
396 of these, namely imagination and sense-perception. He will therefore devote all his energies to distinguishing and examining these three modes of knowing. He will see that there can be no truth or falsity in the strict sense except in the intellect alone, although truth and falsity often originate from the other two modes of knowing; and he will pay careful heed to everything that might deceive him, in order to guard against it. He will make a precise enumeration of all the paths to truth which are open to men, so that he may follow one which is reliable. There are not so many of these that he cannot easily discover them all by means of a sufficient enumeration;[1] this will seem surprising and incredible to the inexperienced. And as soon as he has distinguished, with respect to each individual object, between those items of knowledge which merely fill and adorn the memory and those which really entitle one to be called more learned – an easy task to accomplish . . .[2] he will take the view that any lack of further knowledge on his part is not at all due to any lack of intelligence or method, and that whatever anyone else can know, he too is capable of knowing, if only he properly applies his mind to it. He may often be faced with many questions which this Rule prohibits him from taking up; yet, because he sees clearly that these questions are wholly beyond the reach of the human mind, he will not regard himself as being more ignorant on that account. On the contrary, his very knowing that the matter in question is beyond the bounds of human knowledge will, if he is reasonable, abundantly satisfy his curiosity.

Now, to prevent our being in a state of permanent uncertainty about the powers of the mind, and to prevent our mental labours being misguided and haphazard, we ought once in our life carefully to inquire
397 as to what sort of knowledge human reason is capable of attaining, before we set about acquiring knowledge of things in particular. In order to do this the better, we should, where the objects of inquiry are equally simple, always begin our investigation with those which are more useful.

1 The translation follows the punctuation of A and H here. AT punctuate so as to give the sense '. . . enumeration. What will seem surprising is that as soon as . . .'

2 A lacuna in the texts A, H.

Our method in fact resembles the procedures in the mechanical crafts, which have no need of methods other than their own, and which supply their own instructions for making their own tools. If, for example, someone wanted to practise one of these crafts – to become a blacksmith, say – but did not possess any of the tools, he would be forced at first to use a hard stone (or a rough lump of iron) as an anvil, to make a rock do as a hammer, to make a pair of tongs out of wood, and to put together other such tools as the need arose. Thus prepared, he would not immediately attempt to forge swords, helmets, or other iron implements for others to use; rather he would first of all make hammers, an anvil, tongs and other tools for his own use. What this example shows is that, since in these preliminary inquiries we have managed to discover only some rough precepts which appear to be innate in our minds rather than the product of any skill, we should not immediately try to use these precepts to settle philosophical disputes or to solve mathematical problems. Rather, we should use these precepts in the first instance to seek out with extreme care everything else which is more essential in the investigation of truth, especially since there is no reason why such things should be thought more difficult to discover than any of the solutions to the problems commonly set in geometry, in physics, or in other disciplines.

But the most useful inquiry we can make at this stage is to ask: What is human knowledge and what is its scope? We are at present treating this as one single question, which in our view is the first question of all that 398
should be examined by means of the Rules described above. This is a task which everyone with the slightest love of truth ought to undertake at least once in his life, since the true instruments of knowledge and the entire method are involved in the investigation of the problem. There is, I think, nothing more foolish than presuming, as many do, to argue about the secrets of nature, the influence of the heavens on these lower regions, the prediction of future events, and so on, without ever inquiring whether human reason is adequate for discovering matters such as these. It should not be regarded as an arduous or even difficult task to define the limits of the mental powers we are conscious of possessing, since we often have no hesitation in making judgements about things which are outside us and quite foreign to us. Nor is it an immeasurable task to seek to encompass in thought everything in the universe, with a view to learning in what way particular things may be susceptible of investigation by the human mind. For nothing can be so many-sided or diffuse that it cannot be encompassed within definite limits or arranged under a few headings by means of the method of enumeration we have been discussing. But in order to see how the above points apply to the problem before us, we

shall first divide into two parts whatever is relevant to the question; for the question ought to relate either to us, who have the capacity for knowledge, or to the actual things it is possible to know. We shall discuss these two parts separately.

Within ourselves we are aware that, while it is the intellect alone that is capable of knowledge,[1] it can be helped or hindered by three other faculties, *viz.* imagination, sense-perception, and memory. We must therefore look at these faculties in turn, to see in what respect each of them could be a hindrance, so that we may be on our guard, and in what
399 respect an asset, so that we may make full use of their resources. We shall discuss this part of the question by way of a sufficient enumeration, as the following Rule will make clear.

We should then turn to the things themselves; and we should deal with these only in so far as they are within the reach of the intellect. In that respect we divide them into absolutely simple natures and complex or composite natures. Simple natures must all be either spiritual or corporeal, or belong to each of these categories. As for composite natures, there are some which the intellect experiences as composite before it decides to determine anything about them: but there are others which are put together by the intellect itself. All these points will be explained at greater length in Rule Twelve, where it will be demonstrated that there can be no falsity save in composite natures which are put together by the intellect. In view of this, we divide natures of the latter sort into two further classes, *viz.* those that are deduced from natures which are the most simple and self-evident (which we shall deal with throughout the next book), and those that presuppose others which experience shows us to be composite in reality. We shall reserve the whole of the third book for an account of the latter.[2]

Throughout this treatise we shall try to pursue every humanly accessible path which leads to knowledge of the truth. We shall do this very carefully, and show the paths to be very easy, so that anyone who has mastered the whole method, however mediocre his intelligence, may see
400 that there are no paths closed to him that are open to others, and that his lack of further knowledge is not due to any want of intelligence or method. As often as he applies his mind to acquire knowledge of something, either he will be entirely successful, or at least he will realize that success depends upon some observation which it is not within his power to make – so he will not blame his intelligence, even though he is forced to come to a halt; or, finally, he will be able to demonstrate that the thing he wants to know wholly exceeds the grasp of

1 See footnote on p. 10 above. 2 See Preface, p. 7 above.

the human mind – in which case he will not regard himself as more ignorant on that account, for this discovery amounts to knowledge[1] no less than any other.

Rule Nine

We must concentrate our mind's eye totally upon the most insignificant and easiest of matters, and dwell on them long enough to acquire the habit of intuiting the truth distinctly and clearly.

We have given an account of the two operations of our intellect, intuition and deduction, on which we must, as we said, exclusively rely in our acquisition of knowledge. In this and the following Rule we shall proceed to explain how we can make our employment of intuition and deduction more skilful and at the same time how to cultivate two special mental faculties, *viz.* perspicacity in the distinct intuition of particular things and discernment in the methodical deduction of one thing from another.

We can best learn how mental intuition is to be employed by comparing it with ordinary vision. If one tries to look at many objects at one glance, one sees none of them distinctly. Likewise, if one is inclined 401
to attend to many things at the same time in a single act of thought, one does so with a confused mind. Yet craftsmen who engage in delicate operations, and are used to fixing their eyes on a single point, acquire through practice the ability to make perfect distinctions between things, however minute and delicate. The same is true of those who never let their thinking be distracted by many different objects at the same time, but always devote their whole attention to the simplest and easiest of matters: they become perspicacious.

It is, however, a common failing of mortals to regard what is more difficult as more attractive. Most people consider that they know nothing, even when they see a very clear and simple cause of something; yet at the same time they get carried away with certain sublime and far-fetched arguments of the philosophers, even though these are for the most part based on foundations which no one has ever thoroughly inspected. It is surely madness to think that there is more clarity in darkness than in light. But let us note, those who really do possess knowledge, can discern the truth with equal facility whether thay have derived it from a simple subject or from an obscure one. For once they have hit upon it, they grasp each truth by means of a single and distinct act which is similar in every case. The difference lies entirely in the route followed, which must surely be longer if it leads to a truth which is more remote from completely absolute first principles.

1 Lat. *scientia*; see footnote on p. 10 above.

Everyone ought therefore to acquire the habit of encompassing in his thought at one time facts which are very simple and very few in number –
402 so much so that he never thinks he knows something unless he intuits it just as distinctly as any of the things he knows most distinctly of all. Some people of course are born with a much greater aptitude for this sort of insight than others; but our minds can become much better equipped for it through method and practice. There is, I think, one point above all others which I must stress here, which is that everyone should be firmly convinced that the sciences, however abstruse, are to be deduced only from matters which are easy and highly accessible, and not from those which are grand and obscure.

If, for example, I wish to inquire whether a natural power can travel instantaneously to a distant place, passing through the whole intervening space, I shall not immediately turn my attention to the magnetic force, or the influence of the stars, or even the speed of light, to see whether actions such as these might occur instantaneously; for I would find it more difficult to settle that sort of question than the one at issue. I shall, rather, reflect upon the local motions of bodies, since there can be nothing in this whole area that is more readily perceivable by the senses. And I shall realize that, while a stone cannot pass instantaneously from one place to another, since it is a body, a power similar to the one which moves the stone must be transmitted instantaneously if it is to pass, in its bare state, from one object to another. For instance, if I move one end of a stick, however long it may be, I can easily conceive that the power which moves that part of the stick necessarily moves every other part of it instantaneously, because it is the bare power which is transmitted at that moment, and not the power as it exists in some body, such as a stone which carries it along.[1]

In the same way, if I want to know how one and the same simple cause
403 can give rise simultaneously to opposite effects, I shall not have recourse to the remedies of the physicians, which drive out some humours and keep others in; nor shall I prattle on about the moon's warming things by its light and cooling them by means of some occult quality. Rather, I shall observe a pair of scales, where a single weight raises one scale and lowers the other instantaneously, and similar examples.

Rule Ten

In order to acquire discernment we should exercise our intelligence by investigating what others have already discovered, and methodically

1 Cf. *Optics*, pp. 153–5 below.

survey even the most insignificant products of human skill, especially those which display or presuppose order.

The natural bent of my mind, I confess, is such that the greatest pleasure I have taken in my studies has always come not from accepting the arguments of others but from discovering arguments by my own efforts. It was just this that attracted me to the study of the sciences while I was still in my youth. Whenever the title of a book gave promise of a new discovery, before I read any further I would try and see whether perhaps I could achieve a similar result by means of a certain innate discernment. And I took great care not to deprive myself of this innocent pleasure through a hasty reading of the book. So frequently was I successful in this that eventually I came to realize that I was no longer making my way to the truth of things as others do by way of aimless and blind inquiries, with the aid of luck rather than skill; rather, after many trials I had hit upon some reliable rules of great assistance in finding the truth, and I then used these to devise many more. In this way I carefully elaborated 404
my whole method, and became convinced that the method of study I had pursued from the outset was the most useful of all.

Still, since not all minds have such a natural disposition to puzzle things out by their own exertions, the message of this Rule is that we must not take up the more difficult and arduous issues immediately, but must first tackle the simplest and least exalted arts, and especially those in which order prevails – such as weaving and carpet-making, or the more feminine arts of embroidery, in which threads are interwoven in an infinitely varied pattern. Number-games and any games involving arithmetic, and the like, belong here. It is surprising how much all these activities exercise our minds, provided of course we discover them for ourselves and not from others. For, since nothing in these activities remains hidden and they are totally adapted to human cognitive capacities, they present us in the most distinct way with innumerable instances of order, each one different from the other, yet all regular. Human discernment consists almost entirely in the proper observance of such order.

It was for this reason that we insisted that our inquiries must proceed methodically. In these somewhat trivial subjects the method usually consists simply in constantly following an order, whether it is actually present in the matter in question or is ingeniously read into it. For example, say we want to read something written in an unfamiliar cypher which lacks any apparent order: what we shall do is to invent an order, so as to test every conjecture we can make about individual letters, words, or sentences, and to arrange the characters in such a way that by 405

an enumeration we may discover what can be deduced from them. Above all, we must guard against wasting our time by making random and unmethodical guesses about similarities. Even though problems such as these can often be solved without a method and can sometimes perhaps be solved more quickly through good luck than through method, nevertheless they might dim the light of the mind and make it become so habituated to childish and futile pursuits that thereafter it would always stick to the surface of things and would be unable to penetrate more deeply. But for all that we must not fall into the error of those who occupy their minds exclusively with serious and lofty issues, only to find that after much toil they gain, not the profound science they desired, but mere confusion. We must therefore practise these easier tasks first, and above all methodically, so that by following accessible and familiar paths we may grow accustomed, just as if we were playing a game, to penetrating always to the deeper truth of things. In this way we shall gradually find – much sooner than we might expect – that it is just as easy to deduce, on the basis of evident principles, many propositions which appear very difficult and complicated.

Some will perhaps be surprised that in this context, where we are searching for ways of making ourselves more skilful at deducing some truths on the basis of others, we make no mention of any of the precepts with which dialecticians[1] suppose they govern human reason. They prescribe certain forms of reasoning in which the conclusions follow with such irresistible necessity that if our reason relies on them, even though it
406 takes, as it were, a rest from considering a particular inference clearly and attentively, it can nevertheless draw a conclusion which is certain simply in virtue of the form. But, as we have noticed, truth often slips through these fetters, while those who employ them are left entrapped in them. Others are not so frequently entrapped and, as experience shows, the cleverest sophisms hardly ever deceive anyone who makes use of his untrammelled reason; rather, it is usually the sophists themselves who are led astray.

Our principal concern here is thus to guard against our reason's taking a holiday while we are investigating the truth about some issue; so we reject the forms of reasoning just described as being inimical to our project. Instead we search carefully for everything which may help our mind to stay alert, as we shall show below. But to make it even clearer that the aforementioned art of reasoning contributes nothing whatever to knowledge of the truth, we should realize that, on the basis of their method, dialecticians are unable to formulate a syllogism with a true

1 See footnote, p. 12 above.

conclusion unless they are already in possession of the substance of the conclusion, i.e. unless they have previous knowledge of the very truth deduced in the syllogism. It is obvious therefore that they themselves can learn nothing new from such forms of reasoning, and hence that ordinary dialectic is of no use whatever to those who wish to investigate the truth of things. Its sole advantage is that it sometimes enables us to explain to others arguments which are already known. It should therefore be transferred from philosophy to rhetoric.

Rule Eleven 407

If, after intuiting a number of simple propositions, we deduce something else from them, it is useful to run through them in a continuous and completely uninterrupted train of thought, to reflect on their relations to one another, and to form a distinct and, as far as possible, simultaneous conception of several of them. For in this way our knowledge becomes much more certain, and our mental capacity is enormously increased.

This is a good time to explain more clearly what was said about mental intuition in Rules Three and Seven. In one passage we contrasted it with deduction,[1] and in another only with enumeration,[2] which we defined as an inference drawn from many disconnected facts. But in the same passage we said that a simple deduction of one fact from another is performed by means of intuition.

It was necessary to proceed in that way, because two things are required for mental intuition: first, the proposition intuited must be clear and distinct; second, the whole proposition must be understood all at once, and not bit by bit. But when we think of the process of deduction as we did in Rule Three, it does not seem to take place all at once: inferring one thing from another involves a kind of movement of our mind. In that passage, then, we were justified in distinguishing intuition from deduction. But if we look on deduction as a completed process, as we did in 408
Rule Seven, then it no longer signifies a movement but rather the completion of a movement. That is why we are supposing that the deduction is made through intuition when it is simple and transparent, but not when it is complex and involved. When the latter is the case, we call it 'enumeration' or 'induction', since the intellect cannot simultaneously grasp it as a whole, and its certainty in a sense depends on memory, which must retain the judgements we have made on the individual parts of the enumeration if we are to derive a single conclusion from them taken as a whole.

1 See above, p. 15. 2 See above, p. 25.

All these distinctions had to be made in order to make clear the meaning of this Rule. Rule Nine dealt only with mental intuition; Rule Ten only with enumeration. The present Rule explains the way in which these two operations aid and complement each other; they do this so thoroughly that they seem to coalesce into a single operation, through a movement of thought, as it were, which involves carefully intuiting one thing and passing on at once to the others.

There is, we should point out, a twofold advantage in this fact: it facilitates a more certain knowledge of the conclusion in question, and it makes the mind better able to discover other truths. As we have said, conclusions which embrace more than we can grasp in a single intuition depend for their certainty on memory, and since memory is weak and unstable, it must be refreshed and strengthened through this continuous and repeated movement of thought. Say, for instance, in virtue of several operations, I have discovered the relation between the first and the second magnitude of a series, then the relation between the second and
409 the third and the third and fourth, and lastly the fourth and fifth: that does not necessarily enable me to see what the relation is between the first and the fifth, and I cannot deduce it from the relations I already know unless I remember all of them. That is why it is necessary that I run over them again and again in my mind until I can pass from the first to the last so quickly that memory is left with practically no role to play, and I seem to be intuiting the whole thing at once.

One cannot fail to see that in this way the sluggishness of the mind is redressed and its capacity even enlarged. But in addition we must note that the greatest advantage of this Rule lies in the fact that by reflecting on the mutual dependence of simple propositions we acquire the habit of distinguishing at a glance what is more, and what is less, relative, and by what steps the relative may be reduced to the absolute. For example, if I run through a number of magnitudes which are continued proportionals, I shall be struck by the following points. It is just as easy for me to recognize the relation between the first and the second magnitude, as between the second and the third, the third and fourth, etc., and the act of conceiving is exactly similar in each case. But it is more difficult for me to form a simultaneous conception of the relation of the second magnitude to the first and the third; and it is much more difficult still to conceive the way in which it depends on the first and fourth magnitudes, etc. These considerations enable me to understand why it is that, given only the first and second magnitudes, I can easily find the third and fourth, etc.: the reason is that the discovery is made by means of particular and distinct acts of conceiving. But if only the first and the
410 third are given, it will not be so easy for me to discern the intermediate

magnitude, for this can be done only by means of an act of conceiving which simultaneously involves two of the acts just mentioned. If only the first and the fourth magnitudes are given, it is even more difficult to intuit the two intermediate ones, for in this case three acts of conceiving are simultaneously involved. So, as a logical consequence, it might seem even more difficult to find the three intermediate magnitudes given the first and fifth. Yet this is not the case, owing to a further reason, which is that, although four acts of conceiving are joined together in the present example, they can nevertheless be separated, since four is divisible by another number. So I can obtain the third magnitude alone on the basis of the first and the fifth, then the second on the basis of the first and the third, etc. If one is used to reflecting on these and similar matters, whenever one investigates a new problem one will immediately recognize the source of the difficulty and the simplest method for dealing with it. And that is the greatest aid to knowledge of the truth.

Rule Twelve

Finally we must make use of all the aids which intellect, imagination, sense-perception, and memory afford in order, firstly, to intuit simple propositions distinctly; secondly, to combine[1] *correctly the matters under investigation with what we already know, so that they too may be known; and thirdly, to find out what things should be compared with each other so that we make the most thorough use of all our human powers.*

This Rule sums up everything that has been said above, and sets out a 411
general lesson the details of which remain to be explained as follows.

Where knowledge of things is concerned, only two factors need to be considered: ourselves, the knowing subjects, and the things which are the objects of knowledge. As for ourselves, there are only four faculties which we can use for this purpose, *viz.* intellect, imagination, sense-perception and memory. It is of course only the intellect that is capable of perceiving the truth, but it has to be assisted by imagination, sense-perception and memory if we are not to omit anything which lies within our power. As for the objects of knowledge, it is enough if we examine the following three questions: What presents itself to us spontaneously? How can one thing be known on the basis of something else? What conclusions can be drawn from each of these? This seems to me to be a complete enumeration and to omit nothing which is within the range of human endeavour.

Turning now to the first factor, I should like to explain at this point

1 Reading *componenda*, A; *comparanda*, H, 'to compare'.

what the human mind is, what the body is and how it is informed[1] by the mind, what faculties within the composite whole promote knowledge of things, and what each particular faculty does; but I lack the space, I think, to include all the points which have to be set out before the truth about these matters can be made clear to everyone. For my aim is always to write in such a way that I make no assertions on matters which are apt
412 to give rise to controversy, without first setting out the reasons which led me to make them and which I think others may find convincing too.

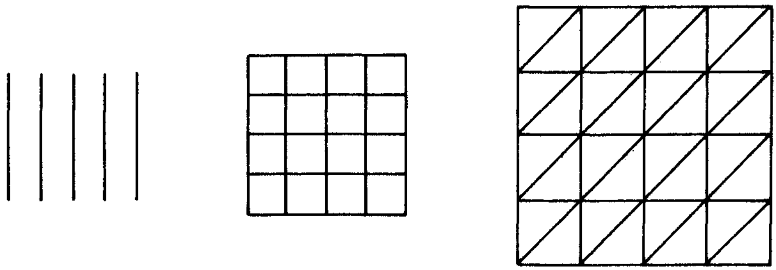
But since I cannot do that here, it will be sufficient if I explain as briefly as possible what, for my purposes, is the most useful way of conceiving everything within us which contributes to our knowledge of things. Of course you are not obliged to believe that things are as I suggest. But what is to prevent you from following these suppositions if it is obvious that they detract not a jot from the truth of things, but simply make everything much clearer? This is just what you do in geometry when you make certain assumptions about quantity, which in no way weaken the force of the demonstrations, even though in physics you often take a different view of the nature of quantity.

Let us then conceive of the matter in the following way. First, in so far as our external senses are all parts of the body, sense-perception, strictly speaking, is merely passive, even though our application of the senses to objects involves action, *viz.* local motion; sense-perception occurs in the same way in which wax takes on an impression from a seal. It should not be thought that I have a mere analogy in mind here: we must think of the external shape of the sentient body as being really changed by the object in exactly the same way as the shape of the surface of the wax is altered by the seal. This is the case, we must admit, not only when we feel some body as having a shape, as being hard or rough to the touch etc., but also when we have a tactile perception of heat or cold and the like. The same is true of the other senses: thus, in the eye, the first opaque membrane receives the shape impressed upon it by multi-coloured light; and in the
413 ears, the nose and the tongue, the first membrane which is impervious to the passage of the object thus takes on a new shape from the sound, the smell and the flavour respectively.

This is a most helpful way of conceiving these matters, since nothing is more readily perceivable by the senses than shape, for it can be touched as well as seen. Moreover, the consequences of this supposition are no more false than those of any other. This is demonstrated by the fact that the concept of shape is so simple and common that it is involved in everything perceivable by the senses. Take colour, for example: whatever

1 A scholastic term conveying the Aristotelian notion that the soul is the 'form' of the body.

you may suppose colour to be, you will not deny that it is extended and consequently has shape. So what troublesome consequences could there be if – while avoiding the useless assumption and pointless invention of some new entity, and without denying what others have preferred to think on the subject – we simply make an abstraction, setting aside every feature of colour apart from its possessing the character of shape, and conceive of the difference between white, blue, red, etc. as being like the difference between the following figures or similar ones?



The same can be said about everything perceivable by the senses, since it is certain that the infinite multiplicity of figures is sufficient for the expression of all the differences in perceptible things.

Secondly, when an external sense organ is stimulated by an object, the 414
figure which it receives is conveyed at one and the same moment to another part of the body known as the 'common' sense,[1] without any entity really passing from the one to the other. In exactly the same way I understand that while I am writing, at the very moment when individual letters are traced on the paper, not only does the point of the pen move, but the slightest motion of this part cannot but be transmitted simultaneously to the whole pen. All these various motions are traced out in the air by the tip of the quill, even though I do not conceive of anything real passing from one end to the other. Who then would think that the connection between the parts of the human body is less close than that between the parts of the pen? What simpler way of portraying the matter can be imagined?

Thirdly, the 'common' sense functions like a seal, fashioning in the phantasy[2] or imagination, as if in wax, the same figures or ideas which come, pure and without body, from the external senses. The phantasy is a

1 An Aristotelian expression signifying an internal sense which receives and co-ordinates impressions from the five external senses. See *De Anima*, III, 1, 425^{a}14.

2 Lat. *phantasia*, a term which for Descartes frequently means the same as *imaginatio*, though it is the term he prefers to use when speaking of the part of the brain in which the physical processes associated with imagining take place. When the latter use is clearly intended, the translation 'corporeal imagination' is adopted below.

genuine part of the body, and is large enough to allow different parts of it to take on many different figures and, generally, to retain them for some time; in which case it is to be identified with what we call 'memory'.

Fourthly, the motive power (i.e. the nerves themselves) has its origin in the brain, where the corporeal imagination is located; and the latter moves the nerves in different ways, just as the 'common' sense is moved by the external senses or the whole pen is moved by its lower end. This
415 example also shows how the corporeal imagination can be the cause of many different movements in the nerves, even though it does not have images of these movements imprinted on it, but has certain other images which enable these movements to follow on. Again, the pen as a whole does not move in exactly the same way as its lower end; on the contrary, the upper part of the pen seems to have a quite different and opposite movement. This enables us to understand how all the movements of other animals can come about, even though we refuse to allow that they have any awareness of things, but merely grant them a purely corporeal imagination. It also enables us to understand how there occur within us all those operations which we perform without any help from reason.

Fifthly, and lastly, the power through which we know things in the strict sense is purely spiritual, and is no less distinct from the whole body than blood is distinct from bone, or the hand from the eye. It is one single power, whether it receives figures from the 'common' sense at the same time as does the corporeal imagination, or applies itself to those which are preserved in the memory, or forms new ones which so preoccupy the imagination that it is often in no position to receive ideas from the 'common' sense at the same time, or to transmit them to the power responsible for motion in accordance with a purely corporeal mode of operation. In all these functions the cognitive power is sometimes passive, sometimes active; sometimes resembling the seal, sometimes the wax. But this should be understood merely as an analogy, for nothing quite like this power is to be found in corporeal things. It is one and the
416 same power: when applying itself along with imagination to the 'common' sense, it is said to see, touch etc.; when addressing itself to the imagination alone, in so far as the latter is invested with various figures, it is said to remember; when applying itself to the imagination in order to form new figures, it is said to imagine or conceive; and lastly, when it acts on its own, it is said to understand. How understanding comes about I shall explain at greater length in the appropriate place. According to its different functions, then, the same power is called either pure intellect, or imagination, or memory, or sense-perception. But when it forms new ideas in the corporeal imagination, or concentrates on those already formed, the proper term for it is 'native intelligence'. We are

regarding it as being capable of performing these different operations; and the distinction between these terms will have to be kept in mind in what follows. If all these matters are conceived along such lines, the attentive reader will have no difficulty in gathering what aids we should seek to obtain from each of these faculties and the lengths to which human endeavour can be stretched in supplementing the shortcomings of our native intelligence.

The intellect can either be stimulated by the imagination or act upon it. Likewise the imagination can act upon the senses through the motive force, by directing them to objects, while the senses in their turn can act upon the imagination, by depicting the images of bodies upon it. But memory is no different from imagination – at least the memory which is corporeal and similar to the one which animals possess. So we can conclude with certainty that when the intellect is concerned with matters in which there is nothing corporeal or similar to the corporeal, it cannot receive any help from those faculties; on the contrary, if it is not to be hampered by them, the senses must be kept back and the imagination must, as far as possible, be divested of every distinct impression. If, however, the intellect proposes to examine something which can be
referred to the body, the idea of that thing must be formed as distinctly as 417
possible in the imagination. In order to do this properly, the thing itself which this idea is to represent should be displayed to the external senses. A plurality of things cannot be of assistance to the intellect in distinctly intuiting individual things. Rather, in order to deduce a single thing from a collection of things – a frequent task – we must discard from the ideas of the things whatever does not demand our present attention, so that the remaining features can be retained more readily in the memory. In the same way, it is not the things themselves which should be displayed to the external senses, but rather certain abbreviated representations of them; and the more compact these are, the handier they are, provided they act as adequate safeguards against lapses of memory. If we observe all these points, then I think we shall omit nothing which pertains to this part of the Rule.

Let us now take up the second factor.[1] Our aim here is to distinguish carefully the notions of simple things from those which are composed of them, and in both cases to try to see where falsity can come in, so that we may guard against it, and to see what can be known with certainty, so that we may concern ourselves exclusively with that. To this end, as before, certain assumptions must be made in this context which perhaps not everyone will accept. But even if they are thought to be no more real than the imaginary circles which astronomers use to describe the

1 I.e. the objects of knowledge. See above, p. 39.

phenomena they study, this matters little, provided they help us to pick out the kind of apprehension of any given thing that may be true and to distinguish it from the kind that may be false.

418 We state our view, then, in the following way. First, when we consider things in the order that corresponds to our knowledge of them, our view of them must be different from what it would be if we were speaking of them in accordance with how they exist in reality. If, for example, we consider some body which has extension and shape, we shall indeed admit that, with respect to the thing itself, it is one single and simple entity. For, viewed in that way, it cannot be said to be a composite made up of corporeal nature, extension and shape, since these constituents have never existed in isolation from each other. Yet with respect to our intellect we call it a composite made up of these three natures, because we understood each of them separately before we were in a position to judge that the three of them are encountered at the same time in one and the same subject. That is why, since we are concerned here with things only in so far as they are perceived by the intellect, we term 'simple' only those things which we know so clearly and distinctly that they cannot be divided by the mind into others which are more distinctly known. Shape, extension and motion, etc. are of this sort; all the rest we conceive to be in a sense composed out of these. This point is to be taken in a very general sense, so that not even the things that we occasionally abstract from these simples are exceptions to it. We are abstracting, for example, when we say that shape is the limit of an extended thing, conceiving by the term 'limit' something more general than shape, since we can talk of the limit of a duration, the limit of a motion, etc. But, even if the sense of the term 'limit' is derived by abstraction from the notion of shape, that is
419 no reason to regard it as simpler than shape. On the contrary, since the term 'limit' is also applied to other things – such as the limit of a duration or a motion, etc., things totally different in kind from shape – it must have been abstracted from these as well. Hence, it is something compounded out of many quite different natures, and the term 'limit' does not have a univocal application in all these cases.

Secondly, those things which are said to be simple with respect to our intellect are, on our view, either purely intellectual or purely material, or common to both. Those simple natures which the intellect recognizes by means of a sort of innate light, without the aid of any corporeal image, are purely intellectual. That there is a number of such things is certain: it is impossible to form any corporeal idea which represents for us what knowledge or doubt or ignorance is, or the action of the will, which may be called 'volition', and the like; and yet we have real knowledge of all of these, knowledge so easy that in order to possess it all we need is some

degree of rationality. Those simple natures, on the other hand, which are recognized to be present only in bodies – such as shape, extension and motion, etc. – are purely material. Lastly, those simples are to be termed 'common' which are ascribed indifferently, now to corporeal things, now to spirits – for instance, existence, unity, duration and the like. To this class we must also refer those common notions which are, as it were, links which connect other simple natures together, and whose self-evidence is the basis for all the rational inferences we make. Examples of these are: 'Things that are the same as a third thing are the same as each other'; 'Things that cannot be related in the same way to a third thing are different in some respect.' These common notions can be known either by the pure intellect or by the intellect as it intuits the images of material 420
things.

Moreover, it is as well to count among the simple natures the corresponding privations and negations, in so far as we understand these. For when I intuit what nothing is, or an instant, or rest, my apprehension is as much genuine knowledge as my understanding what existence is, or duration, or motion. This way of conceiving things will be helpful later on in enabling us to say that all the rest of what we know is put together out of these simple natures. Thus, if I judge that a certain shape is not moving, I shall say that my thought is in some way composed of shape and rest; and similarly in other cases.

Thirdly, these simple natures are all self-evident and never contain any falsity. This can easily be shown if we distinguish between the faculty by which our intellect intuits and knows things and the faculty by which it makes affirmative or negative judgements. For it can happen that we think we are ignorant of things we really know, as for example when we suspect that they contain something else which eludes us, something beyond what we intuit or reach in our thinking, even though we are mistaken in thinking this. For this reason, it is evident that we are mistaken if we ever judge that we lack complete knowledge of any one of these simple natures. For if we have even the slightest grasp of it in our mind – which we surely must have, on the assumption that we are making a judgement about it – it must follow that we have complete knowledge of it. Otherwise it could not be said to be simple, but a 421
composite made up of that which we perceive in it and that of which we judge we are ignorant.

Fourthly, the conjunction between these simple things is either necessary or contingent. The conjunction is necessary when one of them is somehow implied (albeit confusedly) in the concept of the other so that we cannot conceive either of them distinctly if we judge them to be separate from each other. It is in this way that shape is conjoined with

extension, motion with duration or time, etc., because we cannot conceive of a shape which is completely lacking in extension, or a motion wholly lacking in duration. Similarly, if I say that 4 and 3 make 7, the composition is a necessary one, for we do not have a distinct conception of the number 7 unless in a confused sort of way we include 3 and 4 in it. In the same way, whatever we demonstrate concerning figures or numbers necessarily links up with that of which it is affirmed. This necessity applies not just to things which are perceivable by the senses but to others as well. If, for example, Socrates says that he doubts everything, it necessarily follows that he understands at least that he is doubting, and hence that he knows that something can be true or false, etc.; for there is a necessary connection between these facts and the nature of doubt. The union between such things, however, is contingent when the relation conjoining them is not an inseparable one. This is the case when we say that a body is animate, that a man is dressed, etc. Again, there are many instances of things which are necessarily conjoined, even though most people count them as contingent, failing to notice the relation between
422 them: for example the proposition, 'I am, therefore God exists', or 'I understand, therefore I have a mind distinct from my body.' Finally, we must note that very many necessary propositions, when converted, are contingent. Thus from the fact that I exist I may conclude with certainty that God exists, but from the fact that God exists I cannot legitimately assert that I too exist.

Fifthly, it is not possible for us ever to understand anything beyond those simple natures and a certain mixture or compounding of one with another. Indeed, it is often easier to attend at once to several mutually conjoined natures than to separate one of them from the others. For example, I can have knowledge of a triangle, even though it has never occurred to me that this knowledge involves knowledge also of the angle, the line, the number three, shape, extension, etc. But that does not preclude our saying that the nature of a triangle is composed of these other natures and that they are better known than the triangle, for it is just these natures that we understand to be present in it. Perhaps there are many additional natures implicitly contained in the triangle which escape our notice, such as the size of the angles being equal to two right angles, the innumerable relations between the sides and the angles, the size of its surface area, etc.

Sixthly, those natures which we call 'composite' are known by us either because we learn from experience what sort they are, or because we ourselves put them together. Our experience consists of whatever we perceive by means of the senses, whatever we learn from others, and in general whatever reaches our intellect either from external sources or

from its own reflexive self-contemplation. We should note here that the intellect can never be deceived by any experience, provided that when the object is presented to it, it intuits it in a fashion exactly corresponding to the way in which it possesses the object, either within itself or in the imagination. Furthermore, it must not judge that the imagination faithfully represents the objects of the senses, or that the senses take on the true shapes of things, or in short that external things always are just as they appear to be. In all such cases we are liable to go wrong, as we do for example when we take as gospel truth a story which someone has told us; or as someone who has jaundice does when, owing to the yellow tinge of his eyes, he thinks everything is coloured yellow; or again, as we do when our imagination is impaired (as it is in depression) and we think that its disordered images represent real things. But the understanding of the wise man will not be deceived in such cases: while he will judge that whatever comes to him from his imagination really is depicted in it, he will never assert that it passes, complete and unaltered, from the external world to his senses, and from his senses to the corporeal imagination, unless he already has some other grounds for claiming to know this. But whenever we believe that an object of our understanding contains something of which the mind has no immediate perceptual experience, then it is we ourselves who are responsible for its composition. In the same way, when someone who has jaundice is convinced that the things he sees are yellow, this thought of his will be composite, consisting partly of what his corporeal imagination represents to him and partly of the assumption he is making on his own account, *viz.* that the colour looks yellow not owing to any defect of vision but because the things he sees really are yellow. It follows from this that we can go wrong only when we ourselves compose in some way the objects of our belief. 423

Seventhly, this composition can come about in three ways: through impulse, through conjecture or through deduction. It is a case of composition through impulse when, in forming judgements about things, our mind leads us to believe something, not because good reasons convince us of it, but simply because we are caused to believe it, either by some superior power, or by our free will, or by a disposition of the corporeal imagination. The first cause is never a source of error, the second rarely, the third almost always; but the first of these is irrelevant in this context, since it does not come within the scope of method. An example of composition by way of conjecture would be our surmising that above the air there is nothing but a very pure ether, much thinner than air, on the grounds that water, being further from the centre of the globe than earth, is a thinner substance than earth, and air, which rises to greater heights than water, is thinner still. Nothing that we put together 424

in this way really deceives us, so long as we judge it to be merely probable, and never assert it to be true; nor for that matter does it make us any the wiser.[1]

Deduction, therefore, remains as our sole means of compounding things in a way that enables us to be certain of their truth. Yet even with deduction there can be many drawbacks. If, say, we conclude that a given space full of air is empty, on the grounds that we do not perceive anything in it, either by sight, touch, or any other sense, then we are incorrectly conjoining the nature of a vacuum with the nature of this space. This is just what happens when we judge that we can deduce something general and necessary from something particular and contin-
425 gent. But it is within our power to avoid this error, *viz.* by never conjoining things unless we intuit that the conjunction of one with the other is wholly necessary, as we do for example when we deduce that nothing which lacks extension can have a shape, on the grounds that there is a necessary connection between shape and extension, and so on.

From all these considerations we may draw several conclusions. First, we have explained distinctly and, I think, by an adequate enumeration, what at the outset we were able to present only in a confused and rough-and-ready way, *viz.* that there are no paths to certain knowledge of the truth accessible to men save manifest intuition and necessary deduction. We have also explained what the simple natures are which were mentioned in Rule Eight. It is clear that mental intuition extends to all these simple natures and to our knowledge of the necessary connections between them, and in short to everything else which the intellect finds to be present exactly within itself or in the corporeal imagination. But I shall have more to say about deduction below.[2]

Second, we need take no great pains to discover these simple natures, because they are self-evident enough. What requires effort is distinguishing one from another, and intuiting each one separately with steadfast mental gaze. There is no one so dull-witted that he fails to perceive that when sitting he is to some extent different from what he is when standing; but it is not everyone who can distinguish just as
426 distinctly the nature of posture from the other notions which this thought contains, or who can assert that it is only the posture which alters in these two cases. There is good reason for our urging this point here, because the learned are often inclined to be so clever that they find ways of blinding themselves even to facts which are self-evident and which every peasant knows. This is what happens whenever they try and explain

1 Following Crapulli's reading of A and H, *non facit* ('does not make'). AT read *nos facit* ('makes us') without giving any variant reading.

2 See point eight below, p. 50.

things which are self-evident in terms of something even more evident: what they do is to explain something else or nothing at all. For example, can anyone fail to perceive all the respects in which change occurs[1] when we change our place? And when told that 'place is the surface of the surrounding body',[2] would anyone conceive of the matter in the same way? For the surface of the 'surrounding body' can change, even though I do not move or change my place; conversely, it may move along with me, so that, although it still surrounds me, I am no longer in the same place. Again, when people say that motion, something perfectly familiar to everyone, is 'the actuality of a potential being, in so far as it is potential',[3] do they not give the impression of uttering magic words which have a hidden meaning beyond the grasp of the human mind? For who can understand these expressions? Who does not know what motion is? Who would deny that these people are finding a difficulty where none exists? It must be said, then, that we should never explain things of this sort by definitions,[4] in case we take hold of composite things instead of simple ones. Rather, each of us, according to the light of his own mind, must attentively intuit only those things which are 427
distinguished from all others.

Third, the whole of human knowledge[5] consists uniquely in our achieving a distinct perception of how all these simple natures contribute to the composition of other things. This is a very useful point to note, since whenever some difficulty is proposed for investigation, almost everyone gets stuck right at the outset, uncertain as to which thoughts he ought to concentrate his mind on, yet quite convinced that he ought to seek some new kind of entity previously unknown to him. Thus, if the question concerns the nature of the magnet, foreseeing that the topic will prove inaccessible and difficult, he turns his mind away from everything that is evident, and immediately directs it at all the most difficult points, in the vague expectation that by rambling through the barren field of manifold causes he will hit upon something new. But take someone who thinks that nothing in the magnet can be known which does not consist of certain self-evident, simple natures: he is in no doubt about how he should proceed. First he carefully gathers together all the available observations[6] concerning the stone in question; then he tries to deduce from this what sort of mixture of simple natures is necessary for producing all the effects which the magnet is found to have. Once he has

1 Reading *immutatur*, A, H (following Crapulli). AT adopt Leibniz's emendation, *immutamur* ('we change').
2 Cf. Aristotle, *Physics*, IV, 4, 212^{a}5.
3 Cf. Aristotle, *Physics*, III, 1, 201^{a}10.
4 Cf. *Principles*, Part I, art. 10, p. 195 below.
5 Lat. *scientia*; see footnote on p. 10 above.
6 Lat. *experimenta*; see footnote on the equivalent French term *expériences*, p. 143 below.

discovered this mixture, he is in a position to make the bold claim that he has grasped the true nature of the magnet, so far as it is humanly possible to discover it on the basis of given observations.

Lastly, from what has been said it follows that we should not regard some branches of our knowledge of things as more obscure than others,
428 since they are all of the same nature and consist simply in the putting together of self-evident facts. Very few people are aware of this point. Prepossessed by the opposite view, the more confident among them do not hesitate to proclaim their own conjectures as true demonstrations: in matters about which they are completely ignorant they pronounce that they see, as if through a cloud, truths which are often obscure; and they have no qualms about making such claims. They tie their concepts up in various technical terms and, fortified with these, are inclined to discuss, coherently enough, many matters which neither they themselves nor their listeners really understand. But the more modest among them often refrain from investigating many matters – even though they are not difficult and are quite essential for life – simply because they deem themselves unequal to the task. And since they think that such matters can best be understood by others who are more intellectually gifted, they embrace the views of those in whose authority they have more confidence.

Eighthly,[1] deduction can only proceed from words to things, from effects to causes or from causes to effects, from like to like, from parts to parts or to the whole . . .[2]

For the rest, in case anyone should fail to see the interconnection between our Rules, we divide everything that can be known into simple propositions and problems. As for simple propositions, the only rules we provide are those which prepare our cognitive powers for a more distinct intuition of any given object and for a more discerning examination of it. For these simple propositions must occur to us spontaneously; they
429 cannot be sought out. We have covered simple propositions in the preceding twelve Rules, and everything that might in any way facilitate the exercise of reason has, we think, been presented in them. As for problems, however, some can be understood perfectly, even though we do not know the solutions to them, while others are not perfectly understood. We shall deal solely with the former sort of problems in the following twelve Rules, and shall postpone discussion of the latter until

1 Reading *octavo* A, following Crapulli. The variant, *5to*, in H is a misreading of *8to*. This is the eighth 'assumption', following on naturally from the seventh (p. 47 above). Conclusions one to four (pp. 48ff) are a long digression between 'assumptions' seven and eight.

2 There is a lacuna in the texts of A and H at this point. The topic is taken up again below, pp. 53f.

the final set of twelve Rules.[1] The division between perfectly understood and imperfectly understood problems is one that we have introduced quite deliberately: its purpose is partly to save us from having to mention anything which presupposes an acquaintance with what follows, and partly to enable us to set forth first those matters which in our view have to be tackled first if we are to cultivate our mental powers. We must note that a problem is to be counted as perfectly understood only if we have a distinct perception of the following three points: first, what the criteria are which enable us to recognize what we are looking for when we come upon it; second, what exactly is the basis from which we ought to deduce it; third, how it is to be proved that the two are so mutually dependent that the one cannot alter in any respect without there being a corresponding alteration in the other. So now that we possess all the premisses, the only thing that remains to be shown is how the conclusion is to be found. This is not a matter of drawing a single deduction from a single, simple fact, for, as we have already pointed out, that can be done without the aid of rules; it is, rather, a matter of deriving a single fact which depends on many interconnected facts, and of doing this in such a methodical way that no greater intellectual capacity is required than is needed for the simplest inference. Problems of this sort are for the most part abstract, and arise almost exclusively in arithmetic and geometry, which is why 430
they will seem to ignorant people to be of little use. But those who desire a perfect mastery of the latter part of my method (which deals with the other sort of problem) should be advised that a long period of study and practice is needed in order to acquire this technique.

Rule Thirteen

If we perfectly understand a problem we must abstract it from every superfluous conception, reduce it to its simplest terms and, by means of an enumeration, divide it up into the smallest possible parts.

This is the sole respect in which we imitate the dialecticians: when they expound the forms of the syllogisms, they presuppose that the terms or subject-matter of the syllogisms are known; similarly, we are making it a prerequisite here that the problem under investigation is perfectly understood. But we do not distinguish, as they do, a middle term and two extreme terms.[2] We view the whole matter in the following way. First, in every problem there must be something unknown; otherwise there would

1 The final set of twelve Rules was not completed. See Translator's preface, p. 7 above.

2 The middle term of a categorical syllogism is the term which occurs in both premisses but not in the conclusion; the extreme terms are the two terms each of which occurs in one premiss only, and which are connected in the conclusion.

be no point in posing the problem. Secondly, this unknown something must be delineated in some way, otherwise there would be nothing to point us to one line of investigation as opposed to any other. Thirdly, the unknown something can be delineated only by way of something else which is already known. These conditions hold also for imperfect problems. If, for example, the problem concerns the nature of the
431 magnet, we already understand what is meant by the words 'magnet' and 'nature', and it is this knowledge which determines us to adopt one line of inquiry rather than another, etc. But if the problem is to be perfect, we want it to be determinate in every respect, so that we are not looking for anything beyond what can be deduced from the data. For example, someone may ask me what conclusions are to be drawn about the nature of the magnet simply from the experiments which Gilbert claims to have performed, be they true or false.[1] Or again I may be asked to determine what the nature of sound is, solely and precisely from the following data: three strings, A, B and C emit the same sound; B is twice as thick as A, but no longer, and is tensioned by a weight which is twice as heavy: C is twice as long as A, though not so thick, and is tensioned by a weight four times as heavy. It is easy to see from such examples how imperfect problems can all be reduced to perfect ones – as I shall explain at greater length in the appropriate place.[2] We can also see how, by following this Rule, we can abstract a problem which is well understood from every irrelevant conception and reduce it to such a form that we are no longer aware of dealing with this or that subject-matter but only with certain magnitudes in general and the comparison between them.[3] For example, once we have decided to investigate specific observations relating solely to the magnet, we no longer have any difficulty in dismissing all other observations from our mind.

432 Furthermore, the problem should be reduced to the simplest terms according to Rules Five and Six, and it should be divided up according to Rule Seven. Thus if I carry out many observations in my research on the magnet, I shall run through them separately one after another. Again, if the subject of my research is sound, as in the case above, I shall make separate comparisons between strings A and B, then between A and C, etc., with a view to including all of them together in a sufficient enumeration. With respect to the terms of a given problem, these three points are the only ones which the pure intellect has to observe before embarking on the final solution of the problem, for which the following

1 William Gilbert, the English physicist (1540–1603), author of *De Magnete* (1600).
2 Descartes did not complete this task.
3 Reading *comparandas*, H (*componendas*, A, 'composition').

eleven Rules may be required. How this is to be done will be made clearer in the third part of the treatise. By 'problems', moreover, we mean everything in which there lies truth or falsity. We must enumerate the different kinds of problems, so that we may determine what we have the power to achieve in each kind.

As we have already said, there can be no falsity in the mere intuition of things, be they simple or conjoined. In that respect they are not called 'problems'; but they acquire that name as soon as we decide to make a definite judgement about them. Indeed, it is not just the puzzles which others set that we count as problems. Socrates posed a problem about his own ignorance, or rather doubt: when he became aware of his doubt, he began to ask whether it was true that he was in doubt about everything, and his answer was affirmative.

Now, we are seeking to derive things from words, or causes from 433 effects, or effects from causes, or a whole from parts or parts from other parts, or several of these at once.

We say that we are seeking to derive things from words whenever the difficulty lies in the obscurity of the language employed. Riddles all belong to this class of problem: for example the riddle of the Sphinx about the animal which is four-footed to begin with, then two-footed, and later on becomes three-footed; or the one about the anglers standing on the shore with rod and line, maintaining that they no longer have the ones they caught but do have those which they have not yet managed to catch, etc.[1] Moreover, in the vast majority of issues about which the learned dispute, the problem is almost always one of words. There is no need, however, to have such a low opinion of great minds as to think that they have a wrong conception of the things themselves when they fail to explain them in terms which are quite appropriate. When, for example, they define place as 'the surface of the surrounding body', they are not really conceiving anything false, but are merely misusing the word 'place', which in its ordinary use denotes the simple and self-evident nature in virtue of which something is said to be here or there. This nature consists entirely in a certain relation between the thing said to be at the place and the parts of extended[2] space. Some, seeing that the term 'place' has been used to denote the surrounding surface, have improperly called this nature 'intrinsic place',[3] and the same goes for other cases of 434

1 See below, p. 55.
2 Lat. *extensi*, A, H. AT unnecessarily emend to *exterioris* ('exterior'); Crapulli emends to *externi* ('external').
3 In scholastic physics 'intrinsic' place is the space which a body occupies. Cf. *Principles*, pp. 227f below.

this sort. These questions about words arise so frequently that, if philosophers always agreed about the meanings of words, their controversies would almost all be at an end.

It is a problem of deriving causes from effects when in our investigation into something we ask whether it exists, or what it is . . .[1]

Moreover, when we are given a problem to solve, we are often unable to recognize immediately what sort of problem it is, and whether it is a matter of deriving things from words, or causes from effects, etc. Hence it would, I think, be quite pointless to give a lengthy account of the different kinds of problem. It will be less time-consuming and more convenient if instead we make a general and orderly survey of all the points which have to be covered in the solution of any difficulty whatever. Accordingly, no matter what the problem is, we must above all strive to understand distinctly what is being sought.

Frequently people are in such a hurry in their investigation of problems that they set about solving them with their minds blank – without first taking account of the criteria which will enable them to recognize distinctly the thing they are seeking, should they come across it. They are thus behaving like a foolish servant who, sent on some errand by his master, is so eager to obey that he dashes off without instructions and without knowing where he is to go.

In every problem, of course, there has to be something unknown – otherwise the inquiry would be pointless. Nevertheless this unknown
435 something must be delineated by definite conditions, which point us decidedly in one direction of inquiry rather than another. These conditions should, in our view, be gone into right from the very outset. We shall do this if we concentrate our mind's eye on intuiting each individual condition distinctly, looking carefully to see to what extent each condition delimits the unknown object of our inquiry. For in this context the human mind is liable to go wrong in one or other of two ways: it may assume something beyond the data required to define the problem, or on the other hand it may leave something out.

We must take care not to assume more than the data, and not to take the data in too narrow a sense. This is especially true in the case of riddles and other enigmas ingeniously contrived to tax our wits. But it applies sometimes to other problems as well, as for example, when for the sake of a solution we apparently take something as if it were certain, although

1 A lacuna in the texts of A and H. The lost matter is perhaps partially reproduced by Arnauld in the second edition of his *Logic* (the 'Port-Royal Logic'), an extract from which is given in the Appendix below, p. 77.

our confidence in it is due more to ingrained prejudice than to any certain reason. In the riddle of the Sphinx, for example, there is no need to think that the word 'footed' refers exclusively to real feet – to animals' feet. Rather, we should try and see whether it can be applied figuratively to other things as well, as it sometimes is to a baby's hands or an old man's walking-stick, since these are both used, like feet, for getting about. Likewise, in the conundrum about the anglers, we must try not to let the thought of fish so preoccupy our minds that it distracts us from thinking of those tiny creatures which the poor often unwillingly carry about their person and throw away when caught. Again, the question may concern the way in which a certain vessel is constructed, such as the bowl we once saw, which had a column in the centre of it, on top of which was a figure 436
of Tantalus looking as if he was longing to have a drink: water which was poured into the bowl remained within it, as long as the level was below Tantalus' mouth; but as soon as the water reached the unfortunate man's lips, it all ran out. At first glance it might seem that the artistry here lay entirely in the construction of the figure of Tantalus, when in fact that is merely a coincidental feature and by no means a factor which defines the problem. The whole difficulty is this: how must the bowl be constructed if it lets out all the water as soon as, but not before, it reaches a fixed height? One last example: say the question is, 'What can we assert about the motion of the stars, given all the observational data we have relating to them?' In this case we must not freely assume, as the ancients did, that the earth is motionless and fixed at the centre of the universe, just because from our infancy that is how it appeared to us to be. That assumption should be called in doubt so that we may then consider what in the way of certainty our judgement may attain on this matter. And the same goes for other cases of this sort.

On the other hand it is a sin of omission when we fail to take account of some condition necessary for defining a problem, a factor which is either explicitly stated in it or is in some way implied by it. Consider for example the question of perpetual motion – not the natural variety present in the stars and in springs, but the man-made variety. Some have believed that it is possible to achieve perpetual motion of this sort. Regarding the earth as being in perpetual circular motion about its own 437
axis, and the magnet as having all the properties of the earth, they think that they could invent perpetual motion if they could set up a lodestone to move in a circle or at least get it to transfer its motion, along with its other powers, to a piece of iron. Yet even if this were done, what they produced would not be artificial perpetual motion; it would simply be a natural motion which they had harnessed, and would be no different from the continuous motion produced by placing a wheel in a mill-race.

They would therefore be failing to take notice of a condition which is essential for defining the problem, etc.[1]

Once we have sufficiently understood the problem, we should try and see exactly where the difficulty lies, so that by abstracting it from everything else, the problem may be the more easily solved.

In order to find out where the difficulty lies, it is not always sufficient simply to understand the problem; we must also give thought to the particular factors which are essential to it. If any considerations should occur to us which are easy to discover, we shall put these aside; and once these have been eliminated, what we are left with will be just the point we are looking for. Thus in the case described above, it is easy to see how the bowl must be constructed. Once we have set aside, as irrelevant to the issue, such features as the column in the middle, the picture of the bird, etc., the problem is laid bare, which is to explain why it is that all the
438 water flows out of the bowl when it reaches a certain level.

The only thing worth doing, then, in our view is to scrutinize in due order all the factors given in the problem at hand, to dismiss those which we plainly see are irrelevant to the issue, to hold onto those which are essential, and to submit the doubtful ones to a more careful examination.

Rule Fourteen

The problem should be re-expressed in terms of the real extension of bodies and should be pictured in our imagination entirely by means of bare figures. Thus it will be perceived much more distinctly by our intellect.

If, moreover, we are to make use of the imagination as an aid we should note that whenever we deduce something unknown from something already known, it does not follow that we are discovering some new kind of entity, but merely that we are extending our entire knowledge of the topic in question to the point where we perceive that the thing we are looking for participates in this way or that way in the nature of the things given in the statement of the problem. For example, if someone is blind from birth, we should not expect to be able by force of argument to get him to have true ideas of colours just like the ones we have, derived as they are from the senses. But if someone at some time has seen the primary colours, though not the secondary or mixed colours, then by

1 This passage follows the text of A. The text of AT is based on H, and contains several minor emendations.

means of a deduction of sorts it is possible for him to form images even of those he has not seen, in virtue of their similarity to those he has seen.[1] In 439
the same way, if the magnet contains some kind of entity the like of which our intellect has never before perceived, it is pointless to hope that we shall ever get to know it simply by reasoning; in order to do that, we should need to be endowed with some new sense, or with a divine mind. But if we perceive very distinctly that combination of familiar entities or natures which produces the same effects which appear in the magnet, then we shall credit ourselves with having achieved whatever it is possible for the human mind to attain in this matter.

Indeed, it is by means of one and the same idea that we recognize in different subjects[2] each of these familiar entities, such as extension, shape, motion and the like (which we need not enumerate here). The question whether a crown is made of silver or of gold makes no difference to the way we imagine its shape. This common idea is carried over from one subject to the other solely by means of a simple comparison, which enables us to state that the thing we are seeking is in this or that respect similar to, or identical with, or equal to, some given thing. Accordingly, in all reasoning it is only by means of comparison that we attain an exact knowledge of the truth. Consider, for example, the inference: all A is B, all B is C, therefore all A is C. In this case the thing sought and the thing given, A and C, are compared with respect to their both being B, etc. But, as we have frequently insisted,[3] the syllogistic forms are of no help in grasping the truth of things. So it will be to the reader's advantage to 440
reject them altogether and to think of all knowledge whatever – save knowledge obtained through simple and pure intuition of a single, solitary thing – as resulting from a comparison between two or more things. In fact the business of human reason consists almost entirely in preparing for this operation. For when the operation is straightforward and simple, we have no need of a technique to help us intuit the truth which the comparison yields; all we need is the light of nature.

We should note that comparisons are said to be simple and straightforward only when the thing sought and the initial data participate equally in a certain nature. The reason why preparation is required for other sorts of comparison is simply that the common nature in question is not present equally in both, but only by way of other relations or

1 H contains the note (in the margin), 'This example is not absolutely true, but I did not have a better one for explicating what is true.'

2 I.e. subjects of which attributes are predicated – the sense which 'subject' generally bears in the *Rules*.

3 See above, pp. 12, 16, 36.

proportions which imply it. The chief part of human endeavour is simply to reduce these proportions to the point where an equality between what we are seeking and what we already know is clearly visible.

We should note, moreover, that nothing can be reduced to such an equality except what admits of differences of degree, and everything covered by the term 'magnitude'. Consequently, when the terms of a problem have been abstracted from every subject in accordance with the preceding Rule, then we understand that all we have to deal with here are magnitudes in general.

The final point to note is this: if we are to imagine something, and are to make use, not of the pure intellect, but of the intellect aided by images
441 depicted in the imagination, then nothing can be ascribed to magnitudes in general which cannot also be ascribed to any species of magnitude.

It is easy to conclude from this that it will be very useful if we transfer what we understand to hold for magnitudes in general to that species of magnitude which is most readily and distinctly depicted in our imagination. But it follows from what we said in Rule Twelve[1] that this species is the real extension of a body considered in abstraction from everything else about it save its having a shape. In that Rule we conceived of the imagination, along with the ideas existing in it, as being nothing but a real body with a real extension and shape. That indeed is self-evident, since no other subject displays more distinctly all the various differences in proportions. One thing can of course be said to be more or less white than another, one sound more or less sharp than another, and so on; but we cannot determine exactly whether the greater exceeds the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape. Let us then take it as firmly settled that perfectly determinate problems present hardly any difficulty at all, save that of expressing proportions in the form of equalities, and also that everything in which we encounter just this difficulty can easily be, and ought to be, separated from every other subject and then expressed in terms of extension and figures. Accordingly, we shall dismiss everything else from our thoughts and deal exclusively with these until we reach Rule Twenty-five.

442 At this point we should be delighted to come upon a reader favourably disposed towards arithmetic and geometry, though I would rather that he had not yet embarked upon these studies than that he had been taught them in the usual manner. For the Rules which I am about to expound are much more readily employed in the study of these sciences (where they are all that is needed) than in any other sort of problem. Moreover,

1 See above, pp. 40f, 43.

these Rules are so useful in the pursuit of deeper wisdom that I have no hesitation in saying that this part of our method was designed not just for the sake of mathematical problems; our intention was, rather, that the mathematical problems should be studied almost exclusively for the sake of the excellent practice which they give us in the method. I shall not assume anything drawn from the aforementioned disciplines, save perhaps certain facts which are self-evident and accessible to everyone. But the usual sort of knowledge of these subjects which others have, even if not vitiated by any glaring errors, is nevertheless obscured by many vague and ill-conceived principles, which from time to time we shall endeavour to correct in the following pages.

By 'extension' we mean whatever has length, breadth and depth, leaving aside the question whether it is a real body or merely a space. This notion does not, I think, need any further elucidation, for there is nothing more easily perceived by our imagination. Of course the learned often employ distinctions so subtle that they disperse the natural light, and they detect obscurities even in matters which are perfectly clear to peasants. So we must point out to such people that by the term 'extension' we do not mean here something distinct and separate from the subject itself, and that we generally do not recognize philosophical entities of the sort that are not genuinely imaginable. For although someone may convince himself that it is not self-contradictory for 443
extension *per se* to exist all on its own even if everything extended in the universe were annihilated, he would not be employing a corporeal idea in conceiving this, but merely an incorrect judgement of the intellect alone. He will admit this himself if he carefully reflects on the image of extension which he tries to form in his imagination. He will realize that he does not perceive it in isolation from every subject, and that his imagination of it is quite different from his judgement about it. Consequently, whatever our intellect believes about the truth of the matter, these abstract entities are never formed in the imagination in isolation from subjects.

But since henceforth we shall not be undertaking anything without the aid of the imagination, it will be worthwhile to distinguish carefully those ideas by means of which the individual meanings of our words are to be conveyed to our intellect. To this end we suggest for consideration the following three ways of talking: 'Extension occupies a place', 'Body possesses extension', and 'Extension is not body'.

The first sentence shows how 'extension' may be taken to mean 'that which is extended'. Whether I say 'Extension occupies a place' or 'That which is extended occupies a place', my conception is clearly the same in each case. But it does not follow that it is better to use the expression,

'that which is extended', for the sake of avoiding ambiguity, for the latter expression would not convey so distinctly what we are conceiving, *viz.* that some subject occupies a place in virtue of its being extended. The sentence could be taken to mean simply 'That which is extended is a subject occupying a place', with a sense similar to that of the statement 'That which is animate occupies a place.' It is for this reason that we said that here we would be concerned with extension rather than with that which is extended, even though we think that there ought to be no difference in conception between extension and that which is extended.

444 Let us now proceed to the sentence, 'Body possesses extension.' Here we understand the term 'extension' to denote something other than 'body'; yet we do not form two distinct ideas in our imagination, one of extension, the other of body, but just the single idea of extended body. So far as the fact of the matter is concerned I might just as well have said 'Body is extended', or better still 'That which is extended is extended.' This is a peculiarity of those entities which exist only in something else, and which can never be conceived apart from a subject. But when it comes to entities which are really distinguishable from their subjects, the situation is quite different. If, for example, I were to say 'Peter has wealth', my idea of Peter would be quite different from my idea of wealth. Again, if I said 'Paul is wealthy', the content of my imagination would be entirely different from what it would be if I said 'The wealthy man is wealthy.' Many fail to recognize this difference and make the mistake of thinking that extension contains something distinct from that which is extended, in the same way as Paul's wealth is distinct from Paul.

Finally, take the sentence, 'Extension is not body.' The term 'extension' here is understood in a sense quite different from the one above: in this sense there is no specific idea corresponding to it in the imagination. In fact this expression is entirely the work of the pure intellect: it alone has the ability to distinguish between abstract entities of this sort. This is a source of error for many who, not realizing that extension taken in this sense cannot be grasped by the imagination, represent it by means of a real idea. Now such an idea necessarily involves the concept of body. So if they say that extension so conceived is
445 not body, they are unwittingly ensnared into saying 'The same thing is at once body and not body.' It is very important to distinguish utterances in which such terms as 'extension', 'shape', 'number', 'surface', 'line', 'point', 'unity', etc. are given such a narrow sense that they exclude something which is not really distinct from what they signify, as for example in the statements: 'Extension or shape is not body', 'A number is not the thing numbered', 'A surface is the limit of a body', 'A line is the limit of a surface', 'A point is the limit of a line', 'Unity is not a quantity',

etc. All these and similar propositions should be removed completely from the imagination if they are to be true. That is why we shall not be concerned with them in what follows.

We must carefully note the following point with respect to all other propositions in which these terms retain the same meaning and are used in abstraction from subjects, yet do not exclude or deny anything which is not really distinct from what they denote: in these cases we can and should employ the terms with the help of the imagination. For, even if the intellect attends solely and precisely to what the word denotes, the imagination nonetheless ought to form a real idea of the thing, so that the intellect, when required, can be directed towards the other features of the thing which are not conveyed by the term in question, and so that it may never injudiciously take these features to be excluded. Thus, when the problem concerns number, we imagine some subject which is measurable in terms of a set of units. The intellect of course may for the moment confine its attention to this set; nevertheless we must see to it that, in doing so, it does not draw a conclusion which implies that the thing numbered has been excluded from our conception. Those who attribute wonderful and mysterious properties to numbers do just that. They 446
would surely not believe so firmly in such sheer nonsense, if they did not think that number is something distinct from things numbered. Likewise, when we are concerned with a figure, we should bear in mind that we are dealing with an extended subject, conceived simply with respect to its having a shape. When we are concerned with a body, we should bear in mind that it is the same thing we are dealing with, in that it is something which has length, breadth and depth. In the case of a surface, we should conceive of the same thing, as being something with length and breadth – this time leaving out depth, though not denying it. In the case of a line, let us think of it as having just length; and in the case of a point, the same will apply, though this time we should leave out every other property save its being an entity.

Although I am explaining these points at some length here, the minds of mortals are so prejudiced that very few, I fear, are in no danger of losing their way in this area, and most will find that my long discourse gives too brief an account of my meaning. Even arithmetic and geometry lead us astray here in spite of their being the most certain of all the arts. For does not every arithmetician think that numbers are abstracted from every subject by means of the intellect and that they are even to be really distinguished from every subject by means of the imagination? Is there a geometer who does not muddy the manifest clarity of his subject-matter by employing inconsistent principles? The geometer judges that lines have no breadth, surfaces no depth, yet he goes on to construct the one

from the other, not realizing that a line, whose flowing motion he conceives as creating a surface, is a real body, whereas that which lacks breadth is simply a mode of body. But in order not to prolong our account of these matters, it will save time if we explain how we are
447 supposing our object is to be conceived, our aim being to provide the easiest possible demonstration of such truth as may be found in arithmetic and geometry.

In this context, then, we are concerned with an extended object, thinking of it exclusively in terms of its extension, and deliberately refraining from using the term 'quantity'; for there are some philosophers so subtle that they have even distinguished quantity from extension. We are assuming that every problem has been reduced to the point where our sole concern is to discover a certain extension on the basis of a comparison with some other extension which we already know. For in this context we are not expecting to obtain knowledge of any new entity; our intention, rather, is simply to reduce the proportions, however complicated, to the point where we can discover some equality between that which is unknown and something known. Thus it is certain that whatever differences of proportion obtain in other subjects, the same differences can also be found to hold between two or more extensions. Hence it is enough for our purposes if we consider all the characteristics of extension itself which may assist us in elucidating differences in proportion. There are only three such characteristics, *viz.* dimension, unity and shape.

By 'dimension' we mean simply a mode or aspect in respect of which some subject is considered to be measurable. Thus length, breadth and depth are not the only dimensions of a body: weight too is a dimension – the dimension in terms of which objects are weighed. Speed is a dimension – the dimension of motion; and there are countless other
448 instances of this sort. For example, division into several equal parts, whether it be a real or merely intellectual division is, strictly speaking, the dimension in terms of which we count things. The mode which gives rise to number is strictly speaking a species of dimension, though there is some difference between the meanings of the two terms. If we consider the order of the parts in relation to the whole, we are then said to be counting; if on the other hand we regard the whole as being divided up into parts, we are measuring it. For example, we measure centuries in terms of years, days, hours, minutes; if on the other hand we count minutes, hours, days and years, we end up with centuries.

It is clear from this that there can be countless different dimensions within the same subject, that these add absolutely nothing to the things which possess them, and that they are understood in the same way

whether they have a real basis in the objects themselves or are arbitrary inventions of our mind. The weight of a body is something real; so too is the speed of a motion, or the division of a century into years and days; but the division of the day into hours and minutes is not. Yet these all function in the same way from the point of view simply of dimension, which is how they ought to be viewed here and in the mathematical disciplines. Whether dimensions have a real basis is something for the physicists to consider.

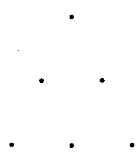
Recognition of this fact throws much light on geometry, for in that discipline almost everyone misconceives the three species of quantity: the line, the surface and the solid. We have already pointed out that the line and the surface are not conceived as being really distinct from the solid or 449
from one another. Indeed, if they are thought of without respect to anything else, as abstractions of the intellect, then they are no more different species of quantity than 'animal' and 'living' in man are different species of substances. We should note incidentally that there is merely a nominal difference between the three dimensions of body – length, breadth and depth; for in any given solid it is quite immaterial which aspect of its extension we take as its length, which as its breadth, etc. Although these three dimensions have a real basis at any rate in every extended thing simply *qua* extended, we are no more concerned with them here than with countless others which are either intellectual fictions or have some other basis in things. Thus in the case of a triangle, if we wish to measure it exactly, there are three real aspects of it which we need to know, *viz.* its three sides, or two sides and one angle, or two angles and its area. Again, in the case of a trapezium there are five factors we need to know; in the case of a tetrahedron, six, etc. These can all be termed 'dimensions'. But if we are to select those dimensions which will be of the greatest assistance to our imagination, we should never attend to more than one or two of them as depicted in our imagination, even though we are well aware that there is an indefinite number involved in the problem at issue. It is part of the method to distinguish as many dimensions as possible, so that, while attending to as few as possible at the same time, we nevertheless proceed to take in all of them one by one.

Unity is the common nature which, we said above,[1] all the things which we are comparing must participate in equally. If no determinate unit is specified in the problem, we may adopt as unit either one of the 450
magnitudes already given or any other magnitude, and this will be the common measure of all the others. We shall regard it as having as many

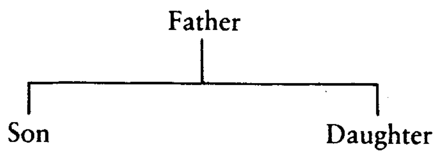
1 See above, pp. 57f.

dimensions as the extreme terms which are to be compared. We shall conceive of it either simply as something extended, abstracting it from everything else – in which case it will be the same as a geometrical point (the movement of which makes up a line, according to the geometers), or as some sort of line, or as a square.

As for figures, we have already shown how ideas of all things can be formed by means of these alone. We have still to point out in this context that, of the innumerable different species of figure, we are to use here only those which most readily express all the various relations or proportions. There are but two kinds of things which are compared with each other: sets[1] and magnitudes. We also have two kinds of figures which we may use to represent these conceptually: for example, the points,



which represent a triangular number;[2] or the diagram which represents someone's family tree,



451 Figures such as these represent sets; while those which are continuous and unbroken, such as △, □ etc., illustrate magnitudes.

Moreover, if we are to explain which of all the available figures we are going to make use of here, we should know that all the relations which may possibly obtain between entities of the same kind should be placed under one or other of two categories, *viz.* order or measure.

We must know, furthermore, that to work out an order is no mean feat, as our method makes clear throughout, that being virtually its entire message. But there is no difficulty whatever in recognizing an order once we have come upon one. By following Rule Seven we can easily survey in our mind the individual parts which we have ordered, because in

1 Lit. 'multitudes' (*multitudines*).
2 Triangular numbers are those which, like 3, 6, 10, 15 etc., can be arranged in the form of a triangle when expressed as a set of points.

relations of this kind the parts are related to one another with respect to themselves alone and not by way of an intermediary third term, as is the case with measures, which it is our sole concern to explicate here. I can recognize what the order between A and B is without considering anything over and above these two terms. But I cannot get to know what the proportion of magnitude between 2 and 3 is without considering some third term, *viz.* the unit which is the common measure of both.

Again, we should realize that, with the aid of the unit we have adopted, 452
it is sometimes possible completely to reduce continuous magnitudes to a set and that this can always be done partially at least. The set of units can then be arranged in such an order that the difficulty involved in discerning a measure becomes simply one of scrutinizing the order. The greatest advantage of our method lies in this progressive ordering.

The final point we should bear in mind is that among the dimensions of a continuous magnitude none is more distinctly conceived than length and breadth, and if we are to compare two different things with each other, we should not attend at the same time to more than these two dimensions in any given figure. For when we have more than two different things to compare, our method demands that we survey them one by one and concentrate on no more than two of them at once.

It is easy to see what conclusions follow from these observations. We have as much reason to abstract propositions from geometrical figures, if the problem has to do with these, as we have from any other subject-matter. The only figures that we need to reserve for this purpose are rectilinear and rectangular surfaces, or straight lines, which we also call figures, because, as we said above,[1] these are just as good as surfaces in assisting us to imagine an object that is really extended. Lastly, these same figures must serve to represent sometimes continuous magnitudes, sometimes a set or a number. To find a simpler way of expressing differences in relation would be beyond the bounds of human endeavour.

Rule Fifteen 453

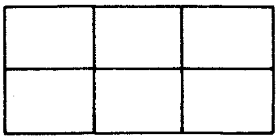
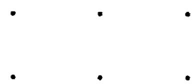
It is generally helpful if we draw these figures and display them before our external senses. In this way it will be easier for us to keep our mind alert.

If we wish to form more distinct images of these figures in our imagination with the aid of a visual display, then it is self-evident how they should be drawn. For example, we shall depict the unit in three ways, *viz.* by means of a square, □, if we think of it only as having length and breadth; by a line, —, if we regard it as having just length; or, lastly,

1 See pp. 61f.

by a point, . , if we view it as the element which goes to make up a set. But however it is depicted and conceived, we shall always understand the unit to be in every sense an extended subject and one susceptible of countless dimensions. The same goes for the terms of the proposition at issue: if we have to attend simultaneously to two different magnitudes belonging to the terms, we shall display them visually as a rectangle, two sides of which will be the two magnitudes in question. If they are incommensurable with the unit, we shall represent them thus:

If commensurable, thus:

 or thus: 

Nothing more is needed, except where the problem concerns a set of
454 units. If, lastly, we are dealing with just one of the magnitudes of the terms, we shall draw a line either in the form of a rectangle, one side of which is the magnitude in question and another is the unit, thus:

(this is what we do when the same line is to be compared with some surface); or simply in the form of a length, thus, ———, if we view it simply as an incommensurable length; or thus, ... , if it is a set.[1]

Rule Sixteen

As for things which do not require the immediate attention of the mind, however necessary they may be for the conclusion, it is better to represent them by very concise symbols rather than by complete figures. It will thus be impossible for our memory to go wrong, and our mind will not be distracted by having to retain these while it is taken up with deducing other matters.

Moreover, as we said,[2] we should not contemplate, in one and the same visual or mental gaze, more than two of the innumerable different

1 The translation of this sentence adheres to the text of A and H (following Crapulli). AT emend the text in such a way that the phrase 'draw a line' becomes 'draw it', and 'when the same line' becomes 'when it'.

2 See above, p. 65.

dimensions which it is possible to depict in the imagination. It is therefore important to retain all the others in such a way that they readily come to mind whenever we need to recall them. It seems that memory has been ordained by nature for this very purpose. But because memory is often unreliable, and in order not to have to squander one jot of our attention on refreshing it while engaged with other thoughts, human ingenuity has given us that happy invention – the practice of writing. Relying on this as an aid, we shall leave absolutely nothing to memory 455
but put down on paper whatever we have to retain, thus allowing the imagination to devote itself freely and completely to the ideas immediately before it. We shall do this by means of very concise symbols, so that after scrutinizing each item (in accordance with Rule Nine), we may be able (in accordance with Rule Eleven) to run through all of them with the swiftest sweep of thought and intuit as many as possible at the same time.

Therefore, whatever is to be viewed as one thing from the point of view of the problem we shall represent by a unique symbol, which can be formed in any way we like. But for the sake of convenience, we shall employ the letters *a*, *b*, *c*, etc. to express magnitudes already known, and *A*, *B*, *C*, etc. for ones that are unknown. To these we shall often prefix the numerals, 1, 2, 3, 4, etc. to indicate how many of them there are; again, we shall also append these as suffixes to indicate the number of the relations which they are to be understood to contain. Thus if I write '$2a^3$', that will mean 'twice the magnitude symbolized by the letter *a*, which contains three relations'. With this device we shall not just be economizing with words but, and this is the important point, we shall also be displaying the terms of the problem in such a pure and naked light that, while nothing useful will be omitted, nothing superfluous will be included – nothing, that is, which might needlessly occupy our mental powers when our mind is having to take in many things at once.

For a clearer understanding of these points, we should note first that arithmeticians usually represent individual magnitudes by means of several units or by some number, whereas in this context we are abstracting just as much from numbers as we did from geometrical figures a little while back[1] – or from any matter whatever. We do this, 456
both to avoid the tedium of long and unnecessary calculation and, most importantly, to see that the parts of the subject relevant to the nature of the problem are kept separate at all times and are not bogged down with pointless numerical expressions. Thus if the problem is to find the hypotenuse of a right-angled triangle whose sides are 9 and 12, the arithmetician will say that it is $\sqrt{225}$ or 15. We on the other hand will

1 See above, p. 63.

substitute a and b for 9 and 12, and will find the hypotenuse to be $\sqrt{a^2+b^2}$, which keeps distinct the two parts a^2 and b^2 which the numerical expression conflates.

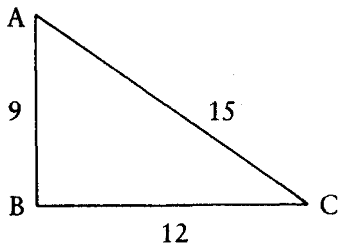
We should note also that those proportions which form a continuing sequence are to be understood in terms of a number of relations; others endeavour to express these proportions in ordinary algebraic terms by means of many dimensions and figures. The first of these they call 'the root', the second 'the square', the third 'the cube', the fourth 'the square of the square'. I confess that I have for a long time been misled by these expressions. For, after the line and the square, nothing, it seemed, could be represented more clearly in my imagination than the cube and other figures modelled on these. Admittedly, I was able to solve many a problem with the help of these. But through long experience I came to realize that by conceiving things in this way I had never discovered anything which I could not have found much more easily and distinctly without it. I realized that such terminology was a source of conceptual confusion and ought to be abandoned completely. For a given magnitude, even though it is called a cube or the square of the square, should never be represented in the imagination otherwise than as a line or a
457 surface, in accordance with the preceding Rule. So we must note above all that the root, the square, the cube, etc. are nothing but magnitudes in continued proportion which, it is always supposed, are preceded by the arbitrary unit mentioned above.[1] The first proportional is related to this unit immediately and by a single relation; the second proportional is related to it by way of the first proportional, and hence by way of two relations; the third proportional by way of the first and the second, and so by way of three relations, etc.[2] From now on, then, the magnitude referred to in algebra as 'the root' we shall term 'the first proportional'; the magnitude referred to as 'the square' we shall call 'the second proportional', and the same goes for the other cases.

Finally, we must note that, even though we are abstracting the terms of a problem from certain numbers in order to investigate its nature, yet it often turns out that the problem can be solved in a simpler way by employing the given numbers than by abstracting from them. This is due to the dual function which numbers have, which is, as we have already mentioned,[3] sometimes to express order, sometimes measure. Accordingly, once we have investigated the problem expressed in general terms, we should re-express it in terms of given numbers, to see whether these

1 Cf. above pp. 63ff.
2 Descartes' idea here is to express, for example, the series, a, a^2, a^3, etc. as $1 \times a$, $a \times a$, $a^2 \times a$, etc.
3 See above, p. 64.

might provide us with a simpler solution. For example, once we have seen that the hypotenuse of a right-angled triangle with sides a and b is $\sqrt{a^2+b^2}$, we should substitute 81 for a^2 and 144 for b^2, the addition of which gives us 225. The root of 225, or the mean proportional between the unit and 225, is 15. We shall see from this that the length of the 458
hypotenuse, 15, is commensurable with the lengths of the other sides, 9 and 12; we shall not generally recognize this from the fact that it is the hypotenuse of a right-angled triangle, two sides of which are in the ratio of 3 to 4. We insist on these distinctions, seeking as we do a knowledge of things that is evident and distinct. The arithmeticians, however, make no such distinctions: they are quite content if the sum they are seeking comes to light, even though they have no idea how it depends on the data; yet that, quite simply, is what knowledge[1] strictly speaking consists in.

But in general we should bear in mind that if we can set it down on paper, we need never commit to memory anything that does not demand our constant attention; otherwise a part of our mind may be distracted by needless recollection from its awareness of the object before it. We ought to write down a list of the terms of the problem as they were stated in the first place; then we should note down the way in which they may be abstracted, and the symbols we might use to represent them. The purpose of this is that once we have found the solution in terms of these symbols, we shall be able to apply it easily to the particular subject we are dealing with, without having recourse to memory. For we always abstract something more general from something less general. So I shall write down the problem in the following way:



The question being to find the hypotenuse, AC, of the right-angled triangle ABC, I first make an abstraction of the problem, so that the question becomes the general one of finding the magnitude of the hypotenuse from the magnitudes of the other sides. I then substitute a for AB, which is 9, and b for BC, which is 12; and so on in other cases.

We should point out that further use will be made of these latter four 459
Rules in the third part of the treatise, though they will be taken in a

1 Lat. *scientia*; see footnote on p. 10 above.

somewhat broader sense than they have been given in the present exposition. But all this will be made clear in due course.

Rule Seventeen

We should make a direct survey of the problem to be solved, disregarding the fact that some of its terms are known and others unknown, and intuiting, through a train of sound reasoning, the dependence of one term on another.

The preceding four Rules have shown us how to abstract determinate and perfectly understood problems from particular subjects and to reduce them to the point where the question becomes simply one of discovering certain magnitudes on the basis of the fact that they bear such and such a relation to certain given magnitudes. Now in the following five Rules we shall explain the method of dealing with these difficulties, so that no matter how many unknown magnitudes there are in a single proposition they can all be arranged in a serial order: the first will stand to the unit as the second to the first, and the third to the second as the fourth to the third, and so on in due sequence. Thus no matter how many of them there are, they will yield a sum equal to some known magnitude. So reliable is our method of doing this that we may safely assert that, however strenuous our efforts, it would be impossible to reduce the magnitudes to simpler terms.

For the present, however, we should note that in every problem to be
460 solved through deduction there is a way of passing from one term to another that is plain and direct: it is the easiest way of all, the others being more difficult and round-about. In order to understand this point we must remember what was said in Rule Eleven, where we explained the nature of that sequence of interlinked propositions which enabled us to see easily,[1] when comparing individual propositions, how the first and the last ones are interrelated, even if we cannot deduce the intermediate ones so easily from the first and last ones. Now if, in order to deduce the way in which the last one depends on the first, we intuit the interdependence between the individual propositions without ever interrupting the order, we are going through the problem in a direct way. If on the other hand we know the first and last propositions to be interconnected in a definite way, and we wish to deduce from this the intermediate ones connecting them, the order we follow will be completely indirect and the reverse of the previous one.[2] Now we are concerned here only with

1 See above, pp. 37f. 2 Cf. above, pp. 24, 38.

complicated questions where the problem is to discern, albeit in a complicated order, certain intermediate propositions, on the basis of our knowledge of the first and last propositions in the series. So the trick here is to treat the unknown ones as if they were known. This may enable us to adopt the easy and direct method of inquiry even in the most complicated of problems. There is no reason why we should not always do this, since from the outset of this part of the treatise[1] our assumption has been that we know that the unknown terms in the problem are so 461
dependent on the known ones that they are wholly determined by them. Accordingly, we shall be carrying out everything this Rule prescribes if, recognizing that the unknown is determined by the known, we reflect on the terms which occur to us first and count the unknown ones among the known, so that by reasoning soundly step by step we may deduce from these all the rest, even the known terms as if they are unknown. We shall postpone illustrating this point (and most of the points to be dealt with below) until Rule Twenty-four:[2] it will be more convenient to expound them there.

Rule Eighteen

For this purpose only four operations are required: addition, subtraction, multiplication and division. The latter two operations should seldom be employed here, for they may lead to needless complication, and they can be carried out more easily later.

A large number of rules is often the result of inexperience in the teacher. Things are much clearer when they are brought under one single general precept rather than split up among many particular ones. For this reason we are bringing under just four heads all the operations needed for working out a problem, i.e. for deducing some magnitudes from others. How it is that these are all we need will become clearer from our account of them.

When we come to know one magnitude on the basis of our prior 462
knowledge of the parts which make it up, the process is one of addition. When we discover a part on the basis of our prior knowledge of the whole and the extent to which the whole exceeds the part, the process is one of subtraction: there is no other possible way of deriving one magnitude from other magnitudes, taken in an absolute sense, which somehow contain it. But if we are to derive some magnitude from others which are quite different from it and which in no way contain it, it is

1 I.e. from Rule Thirteen.
2 The *Rules* in fact end at Rule Twenty-one; see Translator's preface, p. 7 above.

necessary to find some way of relating it to them. If this relation or connection is to be made in a direct way, then we must use multiplication; if in an indirect way, then division.

In order to give a clear account of the latter two operations, we must be apprised of the fact that the unit, which we have spoken about earlier,[1] is here the basis and foundation of all the relations, and occupies the first place in a series of magnitudes which are in continued proportion. The given magnitudes occupy the second place in the series, while those to be discovered occupy the third, fourth, and the remaining places, if the problem in question[2] is a direct one. If, however, the problem is an indirect one, the given magnitude comes last, and the magnitude sought comes in the second place or in other intermediate places.

463 Thus if we are told that as the unit stands to a given magnitude a (5, say), so b (7, say) stands to the number we are seeking, which is ab (i.e. 35), then a and b occupy the second place, and their product, ab, comes in the third place.[3] Again, if we are told that as the unit is to c (e.g. 9), so ab (e.g. 35) is to the number sought, abc (i.e. 315), then abc occupies the fourth place, and is the product of the two multiplications with respect to ab, and c, which occupy the second place;[4] the same holds for other cases. Again, as the unit is to a (i.e. 5), so a is to a^2 (i.e. 25); likewise as the unit is to a (i.e. 5), so a^2 (i.e. 25) is to a^3 (i.e. 125); and lastly as the unit is to a (i.e. 5), so a^3 (i.e. 125) is to a^4 (i.e. 625), etc. For whether a magnitude is multiplied by itself or by a quite different magnitude, the process of multiplication is the same.

If, however, we are told that as the unit is to a given divisor, a (e.g. 5), so the magnitude we are seeking, B (e.g. 7) is to the given dividend, ab (i.e. 35), then the order is confused and indirect,[5] for B can be obtained only by dividing the datum ab by the datum a. The case is the same if the statement is that as the unit is to A (e.g. 5), the number sought, so A is to the datum, a^2 (i.e. 25); or as the unit is to A (5), the number sought, so is A^2 (25) to the datum, a^3 (i.e. 125). And the same is true in other cases. These examples are all included under the term 'division', although we should note that the latter two instances of division are more difficult than the former, since there are more occurrences of the magnitude sought, which therefore involves a greater number of relations. The significance of the latter examples would be the same if we were to employ expressions

464 commonly used by arithmeticians, such as, 'Extract the square root of a^2

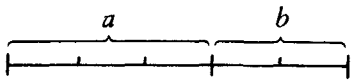
1 Cf. above pp. 63, 68.
2 Reading *propositio*, A, H. AT unnecessarily emend to *proportio* ('proportion').
3 The formula here is $1/a = a/x$. 4 The formula here is $1/c = ab/x$.
5 Henceforth Descartes uses capital letters to denote the unknown magnitudes. See above, p. 67.

(e.g. 25)', or 'Extract the cube root of a^3 (e.g. 125)' – and similarly in the other cases. Alternatively, the problems may be couched in geometrical terms: it amounts to the same thing whether we say 'Find the mean proportional between the arbitrary magnitude called "the unit" and that denoted by the expression a^2', or 'Find two mean proportionals between the unit and a^3', etc.

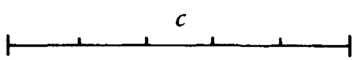
From these considerations it is easy to see how these two operations are all we need for the purpose of discovering whatever magnitudes we are required to deduce from others on the basis of some relation. Once we have understood these operations, the next thing to do is to explain how to present them to the imagination for examination, and how to display them visually, so that later on we may explain their uses or applications.

If addition or subtraction is to be used, we conceive the subject in the form of a line, or in the form of an extended magnitude in which length alone[1] is to be considered. For if we are to add line *a* to line *b*,

we add the one to the other, in the following way,



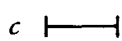
and the result is *c*:



But if the smaller magnitude is to be taken away from the larger, *viz.* *b* 465
from *a*,

we place the one above the other thus:

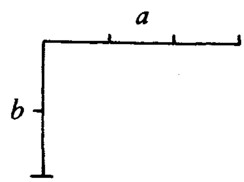
and this will give us that segment of the larger one which the smaller one cannot cover, *viz.*,



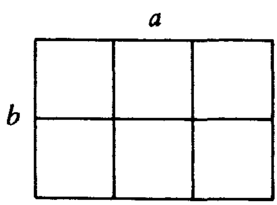
1 Reading *in quâ sola*, H (following Crapulli). AT read *in quâ solâ*. A, 'in which thing alone'.

In multiplication we also conceive the given magnitudes in the form of lines; though in this case we imagine them as forming a rectangle. For if we multiply *a* by *b*,

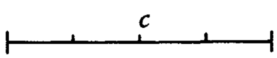
we fit one line at right angles to the other, thus:



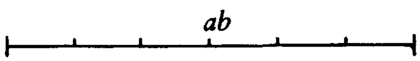
to make the rectangle:



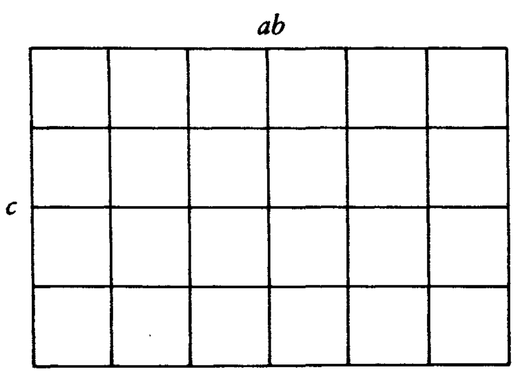
466 Again, if we wish to multiply *ab* by *c*,



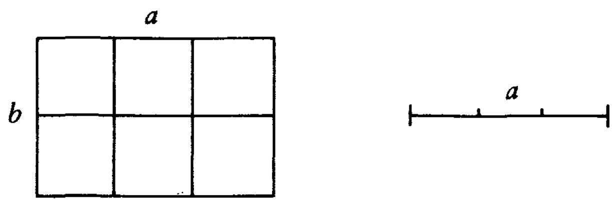
we ought to conceive *ab* as a line, *viz.*,



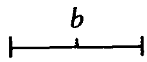
in order to obtain for *abc* the following figure:



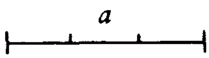
Lastly, in division, where the divisor is given, we imagine the magnitude to be divided as being a rectangle, one side of which is the divisor and the other the quotient. Thus if the rectangle *ab* is to be divided by *a*,



we take away from it the breadth *a*, and are left with *b* as the quotient:



If, on the other hand, we divide the same rectangle by *b*, we take away 467
the height *b*, and the quotient will be *a*:



As for those divisions in which the divisor is not given but only indicated by some relation, as when we are required to extract the square root or the cube root etc., in these cases we must note that the term to be divided, and all the other terms, are always to be conceived as lines which form a series of continued proportionals, the first member of which is the unit, and the last the magnitude to be divided.[1] We shall explain in due course how to find any number of mean proportionals between the latter two magnitudes. For the moment we must be content to point out that we are assuming that we have not yet quite done with these operations, since in order to be performed they require indirect and reverse movements of the imagination, and at present we are dealing only with problems which are to be treated in the direct manner.

As for the other operations, we can easily dispose of these if we conceive them along the lines recommended above. But we have still to show how their terms are initially to be set out. For although, on first dealing with a problem, we are free to conceive of its terms as if they were lines or rectangles, without assigning any other figures to them (as stated in Rule Fourteen),[2] nevertheless in the course of the operation it
frequently turns out that a rectangle, which has been produced by the 468
multiplication of two lines, has to be conceived as a line, for the sake of some further operation. Or again, the same rectangle, or a line resulting

1 Cf. above, pp. 72f. 2 Cf. above, p. 65.

from an addition or subtraction, has to be conceived as a different rectangle drawn above the line which has been designated as its divisor.

It is therefore important to explain here how every rectangle can be transformed into a line, and conversely how a line or even a rectangle can be transformed into another rectangle, one side of which is specified. Geometers can do this very easily, provided they recognize that in comparing lines with some rectangle (as we are now doing), we always conceive the lines as rectangles, one side of which is the length which we adopted as our unit. In this way, the entire business is reduced to the following problem: given a rectangle, to construct upon a given side another rectangle equal to it.

The merest beginner in geometry is of course perfectly familiar with this; nevertheless I want to make the point, in case it should seem that I have omitted something.

Rule Nineteen

Using this method of reasoning, we must try to find as many magnitudes, expressed in two different ways, as there are unknown terms, which we treat as known in order to work out the problem in the direct way. That will give us as many comparisons between two equal terms.[1]

469

Rule Twenty

Once we have found the equations, we must carry out the operations which we have left aside,[2] *never using multiplication when division is in order.*

Rule Twenty-one

If there are many equations of this sort, they should all be reduced to a single one, viz. *to the equation whose terms occupy fewer places in the series of magnitudes which are in continued proportion, i.e. the series in which the order of the terms is to be arranged.*

THE END[3]

1 Descartes' *Geometry* suggests that by the phrase 'expressed in two different ways' he means 'expressed in equations'. The point seems to be that if a problem is to be determinate, there must be as many equations as there are unknowns. Cf. *Geometry*, AT VI 373.

2 I.e. multiplication and division.

3 As far as we know, Descartes did not complete Rules Nineteen to Twenty-one. 'THE END' occurs in A and H.

Appendix

[The extract which follows is from the second edition of Antoine Arnauld and Pierre Nicole's *Logic or the Art of Thinking* (1664).[1] It is known that the authors made use of Descartes' *Rules* when preparing the second edition of their work. In chapter 2 of part 4 they provide a loose paraphrase of the latter part of Rule Thirteen.[2] The paraphrase contains an additional passage which occupies a place corresponding to a lacuna in Rule Thirteen, and it may possibly be a paraphrase of some of the missing material.]

AT X

Problems concerning things can be reduced to four main sorts. 471

In the first sort of problem causes are sought by way of effects. We know, for example, the various effects of the magnet, and we try to find the causes of these effects. We know the various effects which are usually attributed to nature's abhorrence of a vacuum; we inquire whether the latter is the true cause of these effects, and have discovered that it is not.[3] We are familiar with the ebb and flow of the tide, and we want to know what can cause such a great and regular movement.

In the second sort of problem we try to discover effects by way of causes. It has always been known, for example, that wind and water can move bodies with great force; but the ancients did not sufficiently investigate what the effects of these causes could be, and so did not apply them, as they have since been applied in mills, to many things which are very useful to human society and notably ease the burden of human labour, and which ought to be the harvest of true physics. Consequently we can say that the first sort of problem, in which causes are sought by 472
way of their effects, constitutes the entire speculative side of physics, while the second sort of problem, in which effects are sought by way of causes, constitutes the entire practical side.

In the third sort of problem a whole is sought by way of its parts, as for example when we try to find the sum of several numbers by adding them together, or when, given two numbers, we try to find their product by multiplying them together.

In the fourth sort of problem we try to find a part of a whole, given the

1 From ch. 2, part 4, pp. 391ff (AT X 471ff). 2 See above, pp. 54ff.

3 Perhaps a tacit reference to Pascal's *Treatise on the Equilibrium of Liquids and the Weight of the Atmosphere*. Since the latter work was published in 1663 (after Descartes death), some of the content of the above extract (perhaps all) may be due to Arnauld.

whole and some other part of it, as when, given a number and another number to be subtracted from it, we try to find the remainder; or when, given a number, we try to find such and such a part of it.

But in order to extend the scope of the latter two sorts of problem, so that they include what could not properly be brought under the former two sorts, we must note that the word 'part' has to be taken in a very wide sense, as signifying everything that goes to make up a thing – its modes, its extremities, its accidents, its properties, and in general all its attributes. Accordingly we shall be seeking a whole by way of its parts when we try to find the area of a triangle, given its height and its base. On the other hand we shall be seeking a part by way of the whole together with another part when we try to find a side of a rectangle on the basis of our knowledge of its area and one of its sides.